A MODEL FOR THE SELECTION, DESIGN, AND

EVALUATION OF PERFORMANCE MEASURES

by

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ABSTRACT

A matrix representation relating system characteristics to performance measures across a set of activities is presented. Significant uses of the matrix representation are presented which include the selection of measures for implementation, and methodologies for; the comparison of performance measures, the design of performance measures, and the analysis of performance measurement policy. An example from the field of traffic law enforcement is presented and the uses of the matrix representation are illustrated.

KEY WORDS

Performance Measurement
Activity, Characteristic (operator, operand, process)
Surrogate Measure Set
Design of Measures
Coverage Properties
Complementary Measure
Dominant, Inferior Measure
Measure Selection
Policy Augmentation

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1. INTRODUCTION

The validity of any evaluation of activity is dependent on the validity of the measures chosen to characterize performance. Evidence of the importance and growing recognition of performance measurement problems can be seen from the emergence of a number of recent studies (2, 11, 12, 14) which deal specifically with such problems. The emphasis of these studies has been on both qualitative (14) and analytical levels (12). Many analytical approaches to performance measurement attempt to bring to bear variations of utility theory and goal programming as presented in (11) on a specific problem environment. As a result, the methodology developed is often tailored to reflect the focus of the problem context. Since performance measurement problems are of such a diverse nature, a unified generic methodology has been elusive.

Another difficulty facing analytical approaches to performance measurement is the lack of structure in evaluation tasks. The ambiguities and multiple competing objectives familiar to performance measurement applications require enormous flexibility as well as generality from a quantitative methodology for aiding the decision-making process. Examples of typical situations posing difficult problems for performance measurement include evaluation of law enforcement systems, universities, labor unions, research and development activities, and non-profit organizations in general. These problems suggest that a unified, quantitative, generic methodology for performance measurement should provide a uniform means of organizing available information within the problem environment, as well as utilizing it in the decision process.

Recently, a quantitative approach based on matrix representations of performance measurement problems has been developed and embellished by Deutsch and Malmborg (5, 6, 7, 8). The purpose of this paper is to review the developments of this matrix methodology and offer a perspective on where this form of analysis could prove appropriate for modeling performance measurement problems. It will be shown how this technique can offer an orderly means of describing the information available in ill-structured problems, and can provide the mechanism for making such information useful to decision makers. The former is accomplished by reviewing how the model is applied, and the latter is done by reviewing the four uses of the methodology; selecting performance measures, designing performance measures, comparing measure properties, and analyzing measure policies.

In the next section, the matrix model is reviewed and its application is explained. The third section overviews each of the four cited uses of the model using verbal and quantitative definitions. In the final two sections, the use of the model is illustrated by an example from the area of traffic law enforcement, and conclusions are offered.

2. THE MATRIX MODEL

The selection of performance measures is a decision problem of considerable importance to business and government. Measures which are reliable in monitoring variance and conformance with selected criteria in a system's performance are a crucial element for effective management control. One generic approach for evaluating performance measures is the matrix methodology of Deutsch and Malmborg (5).

The matrix model has the advantages of being a straight forward method to utilize, and has the flexibility of being applicable to diverse problem contexts. The method transforms ambiguous evaluation problems into manageable form by requiring the analyst to classify and enumerate the relevant stimuli in a specific problem. Following this procedure, it is possible to perform elementary analysis to the problem which can augment the amount and quality of information which was thought to be available, and use this information to supplement the decision making process. At the very least, application of the matrix methodology forces the analyst to use an organized pattern of thought which; identifies what the key elements in the problem environment are, and which of these elements bear a relationship to each other.

The matrix methodology uses three elements of a system; characteristics, activities, and measures. Characteristics are the subjects upon which performance measurement is focused. The objective in measuring performance is to evaluate characteristics relative to the set of activities. This is accomplished by the implementation of a set of performance measures.

It is important to stress that we use the term characteristic to refer to the subject, object, and outcome in a situation. In a detailed presentation of the matix model (5), possible characteristic components of a problem are subdivided into operator, operand and process subcomponents. The operand refers to the input and/or output of an activity and can assume whatever forms are appropriate to a problem in light of this definition including people, materials, or

information. For example, in a problem of evaluating a new additive for automotive engine oil, operand characteristics could include the oil additive itself (as an input) and perhaps internal operating friction (as an output).

Operator characteristics refer to the object charged with executing a particular task, function or service. Like operand characteristics, operator characteristics assume whatever form is appropriate for a particular problem. In the example evaluating engine oil additives, an obvious operator chracteristic would be the test engines.

Process characteristics describe the objective served through an activity, the manner in which resource inputs are applied by the process, and the information flow within the process. Essentially, process characteristics are characteristic-related but focus on an activity. For example, in the case of evaluating an oil additive, process characteristics might include the rpm at which the engine is run, or the percentage of the additive applied.

2.1 Application of the Matrix Methodology

The matrix methodology proceeds by enumerating the characteristics and activities involved in a problem. In most cases, the objective will be to evaluate the performance of the characteristics in executing the listed activities. The detail with which characteristics and activities are enemerated depends on the needs of the evaluating constituency and the extent to which subdividing the components is possible and/or meaningful.

2.1.1 The Characteristic by Activity Matrix

The construction of the characteristic by activity matrix is initiated by listing the characteristics along a vertical axis and the activities. The second step of the analysis is the specification of the relationship between individual characteristics and activities.

To represent the fact that a relationship exists between a characteristic and an activity, a one is inserted in the cell corresponding to that characteristic and activity. Otherwise, a zero value is used. If a total of C characteristics and A activities have been enumerated, the characteristic by activity matrix will be a C x A 0,1 matrix. An example of a characteristic by activity matrix is shown in Figure 1.

It is important to note that this phase of model implementation does not require any estimate of the degree or nature of the individual characteristic-activity relationship, just whether or not some sort of correlation exists. With this initial matrix construction, it is possible to suggest some preliminary ways of comparing various components. For example, column sums represent the number of characteristics which bear a relationship to individual activities and row sums represent the number of activities over which the performance of individual characteristics can conceivably be measured.

The characteristic by activity matrix can be thought of as a way of describing the physical environment in which performance measurement is to take place. That is, we are attempting to develop or select measures which describe the performance of characteristics which are related to a set of activities in a manner described by the matrix. As will be shown, the characteristic by activity matrix is used to focus the available set of performance measures on the problem at hand.

2.1.2 The Activity by Measure Matrix

A second matrix called the activity by measure matrix, is constructed by enumerating the list of potential performance measures

available for implementation, and defining their relation to the activities with 0,1 elements same way as done with the characteristic by activity matrix. An example of an activity by measure matrix is presented in Figure 2.

2.1.3 The Characteristic by Measure Matrix

Pre-multiplication of the activity by measure matrix by the characteristic by activity matrix results in the characteristic by measure matrix which is a matrix of integer values between 0 and A. This matrix serves as the basis by which performance measurement considerations can be quantified. An example is shown in Figure 3. At this stage, an element within the matrix appears for each characteristic and measure pair. This element represents the number of activities over which a measure and the corresponding characteristic can be associated. We will refer to this association as "coverage" of a characteristic by a measure. That is, a measure monitors or covers a characteristic over the number of activities indicated within the characteristic by measure matrix.

2.2 The Weighted Matrix Model

Up to this point, we have presented the basic framework underlying the matrix methodology. For simplicity, we have not introduced the possibility of differential importance attached to subcomponents within the model. Since it is likely that most applications will represent cases where a varying measure of preference can be associated with activities and measures, it is necessary to embellish the model to accomodate this possibility. This is done in the remainder of this section.

2.2.1 The Assignment of Activity Weights

At this point, the method for generating the characteristic by measure matrix makes no attempt to distinguish between activities. This is equivalent to assuming that identical utility is attached to characteristic-measure relationships over all activities. In many cases, evaluators will have non-identical preferences for the various activities concerned when certain activities are felt to be more important than others for performance measurement purposes. To take account these preferential differences, weights are assigned to the activities. How these activity weights are estimated will depend on the desired accuracy and relationships existing within the problem. Formal procedures weight assignment will be deferred, however, for the moment, we will assume that some means exists for generating these weights.

2.2.2 The Assignment of Measure Weights

As in the case of activities, it is not unreasonable to expect evaluators to have a priori subjective assessments concerning the utility of individual measures. If the set of available measures in such that no preferences can be expressed, the weight of each measure is assumed to be unity.

2.2.3 The Weighted Characteristic by Measure Matrix

Given the weights associated with activities and measures, it becomes possible to generate the corresponding characteristic by measure matrix. The ij^{th} element of the characteristic by measure matrix is,

$$CM_{ik} = \sum_{j=1}^{NACT} \left[(ca_{ij} \cdot wa_j)(am_{jk} \cdot wm_k) \right]$$

where,

NACT = The number of activities.

wa_j = The weight given to activity j.

ca_{ij} = The ij^{th} element of the characteristic by activity matrix, (equals 0 or 1).

wm_j = The weight given to measure j and

am_{ij} = The ij^{th} element of the activity by measure matrix, (equals 0 or 1).

The CM_{ij} elements represent the coverage of characteristics by activities as a multiplicative function of the activity and measure weights associated with characteristic-measure relationships. The unweighted characteristic by measure matrix represents a special case where all measure and activity weights are assumed to be unity. It is also worth noting that we implicitly assume additive utility among characteristic-measure associations across activities. That is, the coverage arising from individual activities remains the same regardless of what combination of activities gives rise to an individual CM_{ij} element. This practice reflects the definition given to activities within the model as distinct components of the process being evaluated. If an activity were thought to be a crucial link within the measurement process, this fact would be reflected in the form of a large weight attached to that activity. If an individual measure failed to provide characteristic coverage of that activity, it would be distinguished by a column of low CM_{ij} elements relative to measures giving coverage to characteristics over that activity.

3. USES OF THE MATRIX METHODOLOGY

Once the characteristic by measure matrix is determined, it can be utilized to assist the decision making process in several distinct ways, including:

1. Selection of performance measures under conditions of scarce implementation resources.
2. Designing new performance measures.
3. Defining properties of measures and measure sets for comparison purposes.
4. Selecting additional measures consistent with a prevailing evaluation policy.

In the sections which follow, the quantitative structures for each application is developed. Following this, an example is presented to illustrate the matrix methodology.

3.1 Selecting a Set of Measures for Implementation

A common concern in performance measurement problems is how to select which measures from the set of available measures should be used. Typically, there is a limit on the resources available for implementing measures and the problem becomes one of finding the "best" subset of a larger set of measures which should be selected.

3.1.1 Some Basic Measure Properties

To assist with this problem, it is useful to define some basic properties of measures arising out of the characteristic by measure matrix. It will be seen from these definitions that performance measures can have differential capabilities in describing the operator, process, or operand characteristics of the activity model. Three absolute delimiters are used to define a measure's coverage of characteristics. To illustrate these, we need to define the following additional notation;

NCHAR – The number of characteristics, and

NMEAS – The number of measures available.

Power: The power of coverage refers to the strength of the relationship between a measure and the total set of characteristics. Power is reflected by the magnitude of the individual elements within a measure column of the characteristic by measure matrix. In terms of the CM_{ij} elements, the power of a measure j can be estimated by:

$$S_j = \sum_{i=1}^{NCHAR} CM_{ij}$$

From the above, we can see that S_j is estimated by taking the column sum corresponding to measure j in the characteristic by measure matrix.

Dimensionality: The dimensionality of coverage refers to the distribution of the elements within a measure column. Dimensionality is thus reflected by the total number of characteristic which are covered (i.e., those with a non-zero relation in the measure column). To define dimensionality in terms of the matrix notation, we define the set of variables Y_i where;

$$Y_i = \begin{cases} 1 \text{ if } CM_{ij} \neq 0 \\ 0 \text{ if } CM_{ij} = 0 \end{cases} \qquad i = 1 \ldots NCHAR$$

The dimensionality of a measure j is then given by;

$$d_j = \sum_{i=1}^{NCHAR} Y_i$$

which represents the number of non-zero elements in the j^{th} column of the characteristic by measure matrix. Dimensionality is a measure of the breadth of coverage of a measure over the total set of characteristics.

Intensity: Intensity refers to the ratio of a measure's power to that measure's dimensionality. It is a measure of the degree to which a measure's total coverage is concentrated on a few characteristics. In terms of the characteristic by measure matrix notation, the measure of intensity for a measure j can be written as;

$$I_j = {}^{S_j}/_{d_j} \ .$$

3.1.2 Additive and Non-Additive Utility Between Measures

The measures of power and intensity represent two criteria for selecting a subset of performance measures for implementation. However, to evaluate measure combinations directly by summing the power (or intensity) of component measures is to assume additive utility between the coverage of measures. Although our definition and weighting procedure for activities circumvents the problem, we cannot arbitarily assume that no interactions exist between measures implemented together. Interactions or "synergy" between the coverage of measures can arise because measures within a combination yield essentially duplicate information (in which case the synergy is negative) and therefore are redundant. Alternatively, individual measures within a combination may complement one another to the extent that the utility stemming from implementing them together exceeds the sum of the utility stemming from implementing each without the other. This would represent a positive synergy.

Two important questions which arise concern how to formulate a measure selection scheme which takes account of synergy between the coverage of measures, and how to estimate these synergistic effects.

3.1.3 Formulation of the Selection Problem

Initially assume that a synergistic coefficient, $\underline{k}^i$, can be associated with each combination of measures. The utility stemming from the implementation of a combination of measures is measured by the product of; the sum of the power of individual measures in the set, and the synergistic coefficient associated with that combination of measures.

To define the measure selection problem, let X_j be a zero-one binary variable where a one indicates measure j is included in a combination of measures, and a zero indicates otherwise. The variables X_j , j=1. . .NMEAS, are defined for each possible combination of measures. For example, suppose there were three candidate measures in a selection problem. For each of the eight possible combinations of measures, we would define the vector of binary variables, θ^i;

$$\theta^1 = \{X_1 = 0, X_2 = 0, X_3 = 0\} \rightarrow \theta^1 = [0,0,0]$$
$$\theta^2 = \{X_1 = 1, X_2 = 0, X_3 = 0\} \rightarrow \theta^2 = [1,0,0]$$
$$\theta^3 = \{X_1 = 0, X_2 = 1, X_3 = 0\} \rightarrow \theta^3 = [0,1,0]$$
$$\theta^4 = \{X_1 = 1, X_2 = 1, X_3 = 0\} \rightarrow \theta^4 = [1,1,0]$$
$$\theta^5 = \{X_1 = 0, X_2 = 0, X_3 = 1\} \rightarrow \theta^5 = [0,0,1]$$
$$\theta^6 = \{X_1 = 1, X_2 = 0, X_3 = 1\} \rightarrow \theta^6 = [1,0,1]$$
$$\theta^7 = \{X_1 = 0, X_2 = 1, X_3 = 1\} \rightarrow \theta^7 = [0,1,1]$$
$$\theta^8 = \{X_1 = 1, X_2 = 1, X_3 = 1\} \rightarrow \theta^8 = [1,1,1]$$

In addition, we define the vectors $\tilde{P}$, and $\mathcal{C}$ as;

$$\tilde{P} = [S_1, S_2, \ldots S_{NMEAS}] \ ,$$

$$\mathcal{C} = [C_1, C_2, \ldots C_{NMEAS}] \ ,$$

where S_j and C_j represent the power and implementation cost of each measure j, respectively. Finally we need to define;

b = the resource budget allocated for the implementation of measures, and

F = the set of θ^i variables.

In general, there will be 2^{NMEAS}, θ^i variables representing the number of possible 0,1 combinations associated with a measure set of size, NMEAS. Similarly, there will be 2^{NMEAS}, K^i, synergistic coefficients.

At this point, we can utilize our definitions to state the measure selection problem as;

$$\max_{i \in F} \{\theta^i \tilde{P}t\} \underline{K^i}$$

$$\text{s.t.} \quad \theta^i C \leq b \quad .$$

The measure selection problem would be solved by determining the θ^{i*}, maximizing the objective function and then implementing those measures having positive (= 1) X_j values within θ^{i*}.

3.1.4 Estimating the Synergistic Coefficients

As we have seen, a coefficient which is intended to capture the synergy between the measures within a set has been defined for each possible measure combination in a problem. How these coefficients are estimated, will depend on how evaluators regard the various attributes of the coverage which results from implementing measures individually or in combination. One possibility would be to favor those measure combinations which brought together measures with the largest CM_{ij} elements, over the widest set of characteristics. Such an approach would focus on the extent to which a group of measures combines those which are best suited for covering specific characteristics, over the entire set of characteristics. For example, if there were four characteristics and a combination of two measures included a measure which concentrated heavily on the first two characteristics and a measure concentrating heavily on the remaining two, the combination would have a high synergistic value. High relative to a measure set consisting of two measures which both concentrate on the first two measures. The latter combination would be considered redundant.

In order to develop an example of an estimation procedure to fit this framework, we need to define some additional attributes of measures stemming from the matrix model. In addition to, WM_k, for each measure, we can define, awt_k, as the activity weight of a measure as;

$$awt_k = \sum_{i=1}^{NCHAR} \sum_{j=1}^{NACT} (ca_{ij} \cdot am_{jk}) wa_j \quad .$$

For each measure, k, awt_k measures the contribution to the coverage of that measure from concentrating on specific activities. If a measure's coverage focuses on activities with a large weight, it will have a correspondingly high activity weight. An alternative means for estimating, awt_k, would be to take the ratio:

$$awt_k = S_k / wm_k \quad ,$$

that is, the ratio of measure power, to measure weight.

Given our definitions of wm_m, awt_m, and S_m for each measure, "m", we can go on to define the following for each combination of measures, i;

$AWT_i = \sum_{m \in i} awt_m$ = the aggregate activity weight for measure combination i.

$\overline{AWT}$ = the mean AWT_i over all possible combinations of measures.

$\text{MWT}_i = \sum_{m \epsilon i} \text{wm}_m$ = the aggregate measure weight for measure combination i.

$\overline{\text{MWT}}$ = the mean MWT_i over all possible combinations of measures.

Given these values for each measure combination, we are now in a position to formulate a means for estimating synergistic effects for measure combinations in accordance with the criteria we have defined. This could be accomplished by defining;

$$\text{SM}_i = \sum_{j=1}^{\text{NCHAR}} \max_{k \epsilon i} \{\text{CM}_{jk}\} = \text{the power of measure}$$

combination, i, defined by taking the largest contribution for each characteristic from among the measures within that combination.

$\overline{\text{SM}}$ = the mean SM_i over all possible combinations of measures. In addition we can define β, (0≤β≤1) as the relative importance attached to activity weights relative to measure weights. The synergistic coefficients for each measure combination, i, is then defined as;

$$\underline{K}^i = (\text{SM}_i / \overline{\text{SM}}) \cdot ((\beta \text{AWT}_i / \overline{\text{AWT}}) + ((1-\beta)\text{MWT}_i / \overline{\text{MWT}}))^{-1} \quad .$$

The above definition would complete our specification of the measure selection problem. It is important, however, to momentarily reflect on the interpretation of our objective criteria. It is essentially seeking that feasible combination of measures bearing relationships to characteristics over the largest number of activities (via the measure of power), subject to finding a combination which bears such a relationship over the widest possible breadth of characteristics. It should be noted that this objective formulation is presented specifically to accomodate these goals and a different objective formulation might be appropriate depending on the goals of an evaluation problem.

In the next section, our analyses based upon the matrix model is widened to define properties useful in comparing measures.

3.2 Comparing the Properties of Performance Measures

In addition to defining the elemental components of measure coverage, i.e., power, dimensionality and intensity, other measure properties are useful in analyses based on the matrix model. The additional properties are based on the power and dimensionality definitions, and are useful for comparing measures within the framework of the matrix model. The definitions given are initially based on the assumption of preferential independence between measures, and relaxation to the more general case of preferential dependence is offered as an extension. The properties of measures and preferentially independent measure sets which are defined include: dominant-inferior, surrogate and complementary.

3.2.1 Dominant and Inferior Measures

Dominant measure. A measure A is said to dominate another measure, B, if their dimensionality is identical with respect to a set of characteristics, and the power of each characteristic by measure element of A is greater than that of B. In terms of the matrix model, this would be expressed as;

$$(\text{CM}_{jB}/\text{CM}_{jA}) \leq 1 \qquad j=1\ldots\text{NCHAR},$$

and

$$\sum_{j=1}^{\text{NCHAR}} \left(\left\lfloor \text{CM}_{jA}/\text{CM}_{jB} \right\rfloor \right) \geq \text{NCHAR} \quad .$$

Inferior Measure. If the power of each characteristic by measure matrix element of measure A is greater than the corresponding element of measure B, then measure B is said to be inferior to measure A over the set of characteristics in question. This property would be expressed as;

$$(CM_{jB}/CM_{jA}) \geq 1 \qquad j=1...NCHAR$$

and,

$$\sum_{j=1}^{NCHAR} \left[CM_{jB}/CM_{jA} \right] \geq NCHAR \ .$$

It is possible to define these properties in terms of characteristics subcomponents such as operator-dominant or process-inferior measures by redefining the right hand side of the above expressions.

3.2.2 Substitutable or "Surrogate" Measures

Surrogate measure. Often it is not feasible due to resource constraints (e.g., time or costs) to implement a given measure. In these cases it is useful to be able to determine an alternative that yields essentially the same coverage over the set of characteristics. We define surrogate measures as those with the same pattern of dimensionality and greater or equal coverage power over all characteristics. Surrogate measures can be surrogate to each other over a single characteristic or groups of characteristics. The most obvious form of surrogates will be over the entire set of characteristics.

In terms of the matrix model, measures A and B would be identical surrogate measures over characteristics 1 through C if they satisfied the relation;

$$\sum_{i=1}^{NCHAR} (CM_{iA} - CM_{iB})^2 = 0 \ .$$

Measure A would also be considered "at least" surrogate to measure B if it satisfied;

$$CM_{iA} - CM_{iB} \geq 0 \qquad i=1...NCHAR \ .$$

3.2.3 Complementary Measures

Retaining the assumption of additive utility, we can define two more useful measure properties in the context of the matrix model. We refer to these as type I and type II complementary measures. Type I are those measures which are reinforcing with the same information. Within the matrix model, type I complementary measures have the same distribution of dimensionality. Two measures A and B are therefore type I complementary measures over characteristics 1 through NCHAR if they satisfy;

$$2\sum_{i=1}^{NCHAR}(Y_{iA}Y_{iB}) = d_A + d_B = 2d_A = 2d_B$$

Type II complementary measures are defined as two measures which are reinforcing with different information. Within the matrix model type II complementary measures are two measures which do not cover the exact same set of characteristics. The measures A and B would be type II complementary measures if they satisfied;

$$2\sum_{i=1}^{NCHAR} Y_{iA}Y_{iB} < d_A + d_B$$

3.2.4 Using the Properties of Measures

Dominant-inferior, surrogate, and complementary measures are useful definitions for comparing individual measures within the matrix framework. They can also be used as a comparison basis for sets of performance measures, if preferential independence among measures can be assumed between individual measures. Three main areas where these measure properties are useful are; finding substitutes for measures which become infeasible for implementation, augmenting currently implemented performance measures with additional measures, and aiding in the selection of performance measures for implem...

3.2.4.1 Finding Substitute Measures

It is reasonable to expect in many situations that performance measures in use at a given time can sometimes become unavailable. This could happen for example, where an increase in the implementation or maintenance cost of a measure changes to an extent where the measure becomes uneconomical. Alternatively, measures may become unavailable as a result of human or equipment failure. In such cases, the matrix methodolgy can be used to determine alternative measures which can be substituted for the measure which becomes unavailable.

In a case such as this, the matrix model can be used to suggest a measure which is at least a surrogate to the lost measure. Candidate measures for the lost measure, ℓ, would be those which satisfy;

$$CM_{i\ell}/CM_{ij} \geq 1 \qquad i=1\ldots NCHAR;\ j\varepsilon\overline{E}$$

where $\overline{E}$ is the set of available measures not currently being implemented. One option for using the above approach would be to select the lowest cost measure satisfying the above.

3.2.4.2 Augmentation of Measure Coverage

In certain circumstances management may choose a set of performance measures for implementation and later determine that they would like to augment this set of measures. Two likely possibilities are; that they would like to augment the set of measures with another measure which provides coverage of the same set of characteristics, or augmentation over at least one characteristic not currently covered.

In the first case, candidate measures would be the set of available, but not implemented measures which are type I complements to the previously implemented set of p measures. That is, those available measures, j, which satisfy the relation;

$$2\sum_{i=1}^{NCHAR}\prod_{k=1}^{p} Y_{ik}Y_{ij} = d_{\overline{p}} + d_j \qquad j\varepsilon\{\overline{E}\}$$

For the case of augmenting a set of p measures with different information, the candidate set would be formed from type II complements, i.e., those measures j which satisfy;

$$\sum_{i=1}^{NCHAR}\prod_{k=1}^{p} Y_{ik}\cdot Y_{ij} < d_{\overline{p}} + d_j \qquad j\varepsilon\{\overline{E}\},$$

where $d_{\overline{p}}$ is the composite dimensionality of the set of p measure.

In effect, the case where the decision maker chooses to augment his set of performance measures with type I complementary measures involves increasing the power and intensity over a fixed set of characteristics. Since the number of characteristics covered does not change, the dimensionality remains fixed but the power and subsequently the intensity of coverage increase. Alternatively, when coverage is augmented with a type II complementary set, the dimensionality as well as the power of coverage are increased. The effect on intensity of coverage in this case is uncertain. Which of these augmentation schemes is preferable depends on the objectives of the individual evaluating constituency.

3.2.4.3 Defining the Efficient Schedule of Measures

Most situations dealing with the selection of performance measures have an analysis phase and a decision making phase. The objective of the analysis stage is to determine the efficient schedule of available measures. This involves eliminating from consideration all available measures dominated by other measures of lower cost. Following this phase, the set of available measures presented to the decision maker does not include measures which are clearly inferior. In this way, the size of the total set of available measures is reduced and the decision making phase is simplified.

To determine the efficient schedule of available performance measures it is necessary to determine the implementation cost of all available measures. We refer to this cost for the i^{th} measure as C_i, for i=1...M. The two possible conditions for eliminating measures from the total set of measures, $\{\underline{M}\}$, which are inferior to some measure j are then,

$$1) \quad \left.\begin{array}{ll} CM_{ik}/CM_{ij} < 1 & i=1\ldots NCHAR \\ \sum_{i=1}^{NCHAR} (CM_{ik}/CM_{ij}) > C_j & \\ \text{and} \quad C_j \leq C_k & \end{array}\right\} \quad j\varepsilon\{\underline{M}\}$$

and for eliminating higher cost exact surrogates,

$$2) \quad \left.\begin{array}{l} \sum_{i=1}^{NCHAR} (CM_{ik} - CM_{ij})^2 = 0 \\ C_k < C_j \end{array}\right\} \quad j\varepsilon\{\underline{M}\}.$$

Measures which adhere to the above criteria and are assigned to the efficient schedule of measures are referred to as efficient measures.

Clearly the criteria for constructing the efficient schedule of measures is only valid for the case of preferential independence among measures. If this assumption did not held, we could use on analogous approach for determining an efficient schedule of combinations of measures which is demonstrated later by an example problem. In any case, the value in determining efficient schedules of measures is a simplification of the measure selection problem by reduction in the total number of available candidate measures.

3.3 The Matrix Model as an Aid in Measure Design

When performance measures become unavailable for implementation and substitutes must be found, it will not always be possible to find such substitutes from the list of currently available measures. In such situations, it becomes necessary to design new measures not previously available. The matrix model can be helpful in suggesting such measures. That is, the model enables the evaluator to identify what activity profiles of measures will give rise to the same form of coverage (within the activity by measure matrix) that was afforded by the measure or measure set which becomes unavailable for implementation. In this way, the zero-one activity profiles suggested by the model are suggestive of where new measures should concentrate their coverage and therefore, are suggestive of how physical information sources (surveys, sample, etc.) should be designed.

3.3.1 Focus of the Design Problem

In utilizing the matrix model as an aid in the design of measures aimed at duplicating the coverage of other measures, the focus is on the characteristic by activity matrix. This is because the characteristic by activity matrix provides a description of how characteristic coverage by measures will be focused. This description can be extended to formulating a set of constraints on the configuration of a measure's zero-one profile in the activity by measure matrix. Once these constraints are defined, alternative activity profiles can be generated which will yield the same coverage results in the characteristic by measure matrix as that measure or measure set which becomes unavailable for implementation.

3.3.2 Constraints Describing Alternative Measure Profiles

To see how the constraints are derived, let A, X, and B represent the characteristic by activity, activity by measure, and characteristic by measure matrices, respectively. The latter can then be written as the constraint:

$$AX = B \quad .$$

Furthermore, assume that all activity and measure weights are one.

For the problem of designing a replacement for a previously available measure which becomes unavailable for implementation, the matrix B can be assumed to be given. To see how alternative 0,1 profiles of alternative performance measures (vectors) can be generated which give rise to the same value of B, consider the following simple example:

$$A = \begin{bmatrix} 1 & 1 & 1 \\ 0 & 1 & 1 \\ 1 & 1 & 0 \end{bmatrix} \quad ; \quad B = \begin{bmatrix} 1 & 1 \\ 2 & 2 \\ 1 & 1 \end{bmatrix}$$

By inspection, we see that X must be a 3 x 2 matrix of the form:

$$X = \begin{bmatrix} X_{11} & X_{12} \\ X_{21} & X_{22} \\ X_{31} & X_{32} \end{bmatrix}$$

The constraint corresponding to this set could therefore be written as:

$$0X_{11} + 2X_{21} + 1X_{31} = 1$$
$$1X_{12} + 2X_{22} + 1X_{32} = 1$$
$$0X_{11} + 2X_{21} + 1X_{31} = 2$$
$$0X_{12} + 2X_{22} + 1X_{32} = 2$$
$$1X_{11} + 2X_{21} + 0X_{31} = 1$$
$$1X_{12} + 2X_{22} + 0X_{32} = 1$$
$$X_{ij} \in S \leftarrow \{\text{binary } 0,1\} \quad \forall_{ij} \quad .$$

It is then possible to find alternative 0,1 combinations of the six X_{ij} variables occurring in the feasible region described above. An important re-formation of the above would find alternatives with at least equivalent coverage by substituting $\geq$ for = in the above constraint set. For the case of designing a single measure profile say $\{X_{11},X_{12},X_{13}\}$, we would substitute the existing vector $\{X_{21},X_{22},X_{23}\}$ zero-one combination, and solve the resulting constraint set for alternative single profiles. If it were reasonable to assume additive utility between measures, we could use the above approach to solve for multiple measure profiles simultaneously.

3.3.3 Ranking Alternative Profiles

An important problem is determining how to rank alternative feasible solutions obtained from the preceeding constraint definition. This question amounts to specifying an appropriate objective function to guide a branch and bound procedure (or other enumeration scheme) through the feasible space. One approach would be to minimize the number of positive X_{ij} values which would tend to produce the solution set with the smallest number of measures. Another approach would be to find the feasible solution with the minimum deviation from some existing measure or measure set. For most cases, however, the greatest flexibility and largest number of profile alternatives are obtained by using a dummy objective function and enumerating the feasible region by simply fathoming only for feasibility. The result will be a variety of profiles (concentrating on different activities) which give rise to the same coverage in the characteristic by measure matrix. In a later section, an example is provided to illustrate the use of this procedure. In most cases, the weight assigned to the measure being designed will be equivalent to the weight of the measure to replace.

3.4 Determining the Implications of a Measure Policy

The matrix methodology can also be applied to decisions aimed at allocating budget increases for performance measurement when no clear definitions of optimality can be discerned other than consistency with prevailing policy and extremely basic economic considerations. That is, the model can be used to infer a functional evaluation criteria from existing performance measurement policies.

This is a potentially important problem area since the context of individual analyses of performance measurement problems is so variable, it is difficult to develop normative performance measurement policies for ill structured problems that are valid in any general sense. Even though it is not possible to identify a generally preferred measure policy, using the matrix model does allow quantification of the differences between policies. This enables the evaluator at least to determine those measures and measure sets which are consistent in their characteristic coverage policies.

Essentially, the technique proceeds by describing an existing performance measurement policy in a compact form. From this form, an objective criteria for selecting among combinations of available measures to achieve consistency with the prevailing policy is developed. The basic approach used is to identify the emphasis of a measure set in terms of characteristic coverage for the cases of; cost information available, and cost information unavailable. This approach can be used alone or augmented with another procedure for taking account of the relative coverage costs of individual characteristics. To form

these policy descriptions, it is first necessary to calculate average "per activity" coverage costs for individual measures and characteristics.

3.4.1 Calculation of Characteristic and Measure Average Per Activity Costs

We will initially make the assumptions that the utility stemming from covering individual characteristics is equal for all characteristics for purposes of determining coverage economy between them, and preferential independence exists between measures. Under these assumptions, a way to determine the cost efficiency of a set of measures (if implementation costs are available) is to calculate the price paid for covering each individual characteristic. This figure can then be reviewed against the overall average price for covering each individual characteristic. In order to calculate the average price for covering a characteristic over an activity, the average cost per activity is calculated for each measure. This figure is then average across measures for each characteristic. The result is the average price of covering each characteristic over one activity.

For example, suppose we had three characteristics (a), (b) and (c) covered by three measures (1), (2) and (3), and the following characteristic by measure matrix:

$$\begin{array}{c} (a) \\ (b) \\ (c) \\ \\ \end{array} \begin{array}{c} \begin{bmatrix} 2 & 0 & 1 \\ 1 & 3 & 1 \\ 1 & 0 & 1 \end{bmatrix} \\ (1)\ (2)\ (3) \end{array}$$

The implementation costs for measures (1), (2) and (3) are 150, 200, and 100 dollars, respectively. The average cost per activity of coverage by measure (1) is 37.5 (150/4), by measure (2) is 66.67 (200/3), and by measure (3) is 33.33 (100/3). For characteristic (a), two activities are covered by measure (1) at a cost of 37.5 each, and one activity at 33.33 is covered by measure (3). The average cost per activity for covering characteristic (a) is therefore 36.11. Similarly, the average cost per activity of covering characteristics (b) and (c) is 54.17 and 35.42, respectively.

3.4.2 Spending Emphasis of a Policy

A measure set or "implementation policy" can be described by the relative spending emphasis directed toward each characteristic. For instance, referring to the simple example just presented, if a policy were to implement measure (1) alone, (at a cost of $150), we would be spending 50% of our resources to cover characteristic (a), and 25% each to cover characteristic (b) and (c). The cost per activity would be $37.5 for each characteristic. With this criteria, the policy of implementing measure (1) alone could be described by the equation;

$$.5(a) + .25(b) + .25(c) \quad .$$

If implementation costs on individual measures are not available, a description of a policy implied by a set of performance measures can be based on the relative activity weight or number of activities covered for each characteristic. If the measures (1) and (2) from the example were implemented, we would merely calculate the ratio of the number of activities covered for each characteristic to the total number of activities covered. Using this criteria, we can describe the policy of implementing measures (1) and (2) using no cost information by the equation;

$$.2857(a) + .5714(b) + .1429(c) \quad .$$

3.4.3 The Relative Economy of a Measure Policy

For cases where cost information is available, it is possible to account for cost considerations in the selection of measures. One way to do this is to compare the per activity cost of covering each characteristic with per activity costs over all available measures. The ratio of these costs for each characteristic under each alternative, provides a measure of the relative cost efficiency obtained in implementing a particular measure set. We can again use the earlier example for illustration. For the alternative of implementing measure (1) alone, the per activity costs of covering characteristics (a), (b), and (c) are all 37.5. The per activity costs over all available measures are; 36.11 for characteristic (a), 54.17 for characteristic (b), and 35.42 for characteristic (c). Taking the ratio of overall costs to the costs stemming from implementation of measure (1) alone yields the equation;

$$.9629(a) + 1.4445(b) + .9445(c) \quad .$$

Normalization of the above equation yields;

$$.2873(a) + .4309(b) + .2818(c) \quad .$$

For the case where covering any characteristic yields the same utility as covering any other characteristic, we could use the above equation as an aid in selecting among measures. This equation would suggest that the prevailing policy of implementing measure (1) alone realizes the greatest economy in coverage of characteristic (b). As a result, if our objective in selecting among new measures is consistency with the prevailing policy, the above would serve as a guide in the measure selection decision.

3.4.4 Developing a Composite Policy Description

Up to this point we have developed equations describing the spending emphasis and implied economy for describing a measure policy. These equations can be combined by specification of a weight, α, which measures the extent to which the evaluating constitutency considers spending emphasis as more indicative of the utility embodied in a measure policy than the implied economy. If we let X_i represent the i^{th} coefficient of the spending emphasis equation and X_i' be the i^{th} coefficient of the implied economy equation, the equations over the characteristics one through NCHAR can be combined by the equation;

$$\sum_{i=1}^{NCHAR} (\alpha X_i + (1-\alpha)X_i') \quad .$$

From our earlier example, if we assume the equally weighted case, i.e., $\alpha = .5$, we obtain the descriptive equation;

$$.1437(a) + .1077(b) + .0707(c) \quad .$$

Normalization yeilds;

$$.4464(a) + .3346(b) + .2190(c) \quad .$$

3.4.5 Using the Descriptive Equations of a Measure Policy

This composite equation can be used in the measure selection problem (where the objective is consistency with prevailing policy) in two ways. The first way is to define the set of all feasible combinations of available measures and use the equation and the elements of the characteristic by measure matrix to compute an objective value for each feasible measure combination. Measure combinations could then be ranked in the order of this objective value for selection.

The alternative way of using the composite equation in measure selection would be to use the characteristic by measure elements and the equation to compute an objective coefficient for each available measure individually. These coefficients could then be used in an optimization problem where a binary variable represents each measure and the model selects a set of one or more measures.

In the section which follows, an example is presented which will illustrate the use of the matrix methodology in performance measurement. The problem is defined in the context of a local government attempting to evaluate the performance of its overall traffic law enforcement function. The problem parameters are defined and some methodologies based on the matrix model are applied to illustrate the flexibility and possible uses of the model.

4. AN EXAMPLE FROM TRAFFIC LAW ENFORCEMENT

To illustrate possible applications of the matrix model consider a county government attempting to evaluate local traffic safety and traffic law enforcement. The list of potential measures which are initially proposed for monitoring the performance of local traffic law enforcement include:

- An analysis of revenues from traffic court fines which breaks down the revenue stemming from each violation type. The measure could be used to monitor the extent to which each of several enforcement bodies were active in issuing traffic citations.
- An investigation of previous and future auto accidents to determine where violations were likely to be involved, and if citations or arrests resulted in such cases.

- A study to determine the actual probability of detection and subsequent issuance of citations, etc., for violations in various sectors of the jurisdiction.
- A profile of repeat offenders in traffic violation cases to detect if a pattern exists in the majority of traffic court cases and determine areas of concentration or potential discrimination in enforcement.

Within the jurisdiction, three distinct enforcement bodies present citations to a single local traffic court, and one emergency services organization (ambulance, fire, etc.) exists. As a result, five characteristics are considered in the analysis:

(a.) city police traffic enforcement

(b.) county police traffic enforcement

(c.) county emergency services

(d.) interstate highway radar patrol by state police

(e.) traffic court.

In the overall enforcement process, five activities have been identified as relevant for evaluating the system's performance. These activities and their associated weights are;

1. the issuance of traffic citations, $wa_1 = 5$.
2. the testing of offender blood alchohol levels, $wa_2 = 2$.
3. the arrest and detention of offenders for serious violations, $wa_3 = 3$.
4. emergency medical treatment provided at the scene of accidents, $wa_4 = 3$.
5. resolution of cases submitted to traffic court including fines and other sentencing, $wa_5 = 4$.

Local administrators familiar with the jurisdiction and the function of the various organizations within it were used to specify the 0,1 relationships of the characteristic by activity matrix presented in Table 1. Although correlations were known to exist between activities, (e.g., arrests were nearly always accompanied by citations) it was believed that activities were defined such that the information content from covering individual activities was independent for the purposes of this evaluation.

The parties suggesting each of the four measures have specified the zero-one relationships of the activity by measure matrix presented in Table 2. Pre-multiplication of the characteristic by activity and the activity by measure matrices gives the following unweighted characteristic by measure matrix:

CHARACTERISTICS	fines study ①	accident investigation ②	detection probability ③	repeat offender study ④
city police (a)	1	3	1	2
county police (b)	1	2	2	1
emergency services (c)	0	1	0	1
highway radar patrol (d)	1	1	1	0
traffic court (e)	1	1	0	1

MEASURES

CHARACTERISTICS	citation writing	blood alchocol testing	arrest/ detention	emergency medical care	resolution of court cases
city police (a)	1	1	1	0	0
county police (b)	1	0	1	0	0
emergency services (c)	0	1	0	1	0
highway radar patrol (d)	1	0	0	1	0
traffic court (e)	0	0	0	0	1

ACTIVITIES

Table 1. Characteristic by Activity Matrix for the Traffic Law Enforcement Example

ACTIVITIES	fines study	accident investigations	detection probability	repeat offender study
citation writing	1	1	1	0
blood alchohol testing	0	1	0	1
arrest/detention	0	1	1	1
emergency medical care	0	0	0	0
resolution of court cases	1	1	0	1

MEASURES

Table 2. Activity by Measure Matrix for the Traffic Law Enforcement Example

Associated with the available measures were the following implementation costs, and measure weights

- The study of traffic court fines and sentencing would involve compiling and key punching of items derived from court records. Overtime costs for the clerical staff, key punching and data processing for this measure is estimated to cost $1,200 with a weight, $wm_1 = 4$.
- The rigorous investigative effort required for implementation of the second measure will involve time spent by detective and labratory personnel worth an estimated $5,000 and represents a weight, $wm_2 = 3$.
- To determine the actual probability of detection and citation for violations would require a carefully planned program measuring police visibility and response. Since some degree of private consulting services is involved, the cost is predicted to reach $4,000 and represents a weight, $wm_3 = 4$.

Assuming a measure implementation budget of $5,000, we can proceed to set up the problem of selecting a measure set. This is done by defining the θ_i vector variables representing all possible measure combinations, and calculating the associated synergistic coefficients, $\underline{K}^i$. These values are presented in Table 3.

- The study of repeat offender profiles would involve little data processing since police records on such individuals are already compiled, but actual analysis for a sufficient number of offenders is estimated to cost $2,500, and represents a weight, $wm_4 = 3$.

4.1 Selecting a Measure Set for Implementation

The weighted characteristic by measure matrix which results is the following;

CHARACTERISTICS	① fines study	② accident investigation	③ detection probability	④ repeat offender study
city police (a)	20	30	32	15
county police (b)	20	24	32	9
emergency services (c)	0	6	0	6
highway radar patrol (d)	20	15	20	0
traffic court (e)	16	12	0	12

The θ^i vector variables in Table 3 comprise the set F. The power of coverage for each measure is;

$$\left.\begin{array}{l} S_1 = 76 \\ S_2 = 81 \\ S_3 = 84 \\ S_4 = 42 \end{array}\right\} \quad \tilde{P} = [76, 81, 84, 42]$$

The measure selection problem would be written as;

$$\max_{i \in F} \{\theta^i [76, 81, 84, 42]\} \underline{K}^i$$

$\theta_1 = [0, 0, 0, 0]$; $\underline{K}^1 = 0$

$\theta_2 = [1, 0, 0, 0]$; $\underline{K}^2 = 1.72$

$\theta_3 = [0, 1, 0, 0]$; $\underline{K}^3 = 1.80$

$\theta_4 = [1, 1, 0, 0]$; $\underline{K}^4 = 1.04$

$\theta_5 = [0, 0, 1, 0]$; $\underline{K}^5 = 1.82$

$\theta_6 = [1, 0, 1, 0]$; $\underline{K}^6 = 1.11$

$\theta_7 = [0, 1, 1, 0]$; $\underline{K}^7 = 1.08$

$\theta_8 = [1, 1, 1, 0]$; $\underline{K}^8 = 0.76$

$\theta_9 = [0, 0, 0, 1]$; $\underline{K}^9 = 1.28$

$\theta_{10} = [1, 0, 0, 1]$; $\underline{K}^{10} = 1.06$

$\theta_{11} = [0, 1, 0, 1]$; $\underline{K}^{11} = 1.07$

$\theta_{12} = [1, 1, 0, 1]$; $\underline{K}^{12} = 0.77$

$\theta_{13} = [0, 0, 1, 1]$; $\underline{K}^{13} = 1.29$

$\theta_{14} = [1, 0, 1, 1]$; $\underline{K}^{14} = 0.86$

$\theta_{15} = [0, 1, 1, 1]$; $\underline{K}^{15} = 0.80$

$\theta_{16} = [1, 1, 1, 1]$; $\underline{K}^{16} = 0.62$

Table 3. The θ^i Vector Variables and Computed Synergistic Coefficients

subject to $$\theta^i \begin{bmatrix} 1200 \\ 5000 \\ 4000 \\ 2500 \end{bmatrix} \leq 5000$$

Solving the above yields that the optimal solution is for i=5. Since $\theta^5 = [0, 0, 1, 0]$, the optimal policy is to implement measure 3. That is, proceed with the study to determine the actual probability of detection for violators.

4.2 Determination of Available Measure Substitutes

To illustrate how the matrix model can be used in finding substitute measures. Assume that we had decided to implement measure 2 but it is found that 2 cannot be implemented for lack of available manpower. Rather than repeating an overall selection procedure, we can determine the lowest cost surrogate for the lost measure. From examination of the characteristic by measure matrix, it is apparent that no single measure is an adequate substitute (i.e., surrogate) to measure 2. Consequently, a group of measures must be combined to approximate the coverage of measure 2.

The first possibility which comes to mind is the combination of measures 1 and 4 at a cost of only 3,700 (1,200 + 2,500). However, it can be seen from the ISM that these measures are up to 30% redundant in their coverage. As a result, together they do not represent a surrogate to measure 2. The other possibility is to combine measures 3 and 4 which have an intermeasures synergistic effect of positive 10%. Although the total implementation cost of 6,500 exceeds other possible combinations, it is the only measure combination which is at least surrogate to measure 2 when intermeasure synergistic effects are considered.

For the example presented, no single measure or measure combination is dominated by a lower cost measure or measure combination when intermeasure synergistic effects are considered. Therefore, the efficient schedule of measures includes all possible measure combinations. In cases such as this, the number of measure sets which are candidates for implementation cannot be reduced by determining the efficient schedule of measures.

4.3 Analysis of the Implications of the Selected Policy

If it were determined that our originally selected measure (2) could be implemented, we could use this information to infer the implications of this policy. The first step is to compute the average cost per activity for each measure;

cost per activity (1) = $^{1200}/_4 = 300$
" " " (2) = $^{5000}/_8 = 625$
" " " (3) = $^{4000}/_4 = 1000$
" " " (4) = $^{2500}/_5 = 500$

This information and the characteristic by measure matrix is then used to compute the average cost of covering an activity for each characteristic:

cost per activity of (a) = $\{1(300) + 3(625) + 1(1000) + 2(500)\}/7 = 507.14$
" " " " (b) = $\{1(300) + 2(625) + 2(1000) + 1(500)\}/6 = 675.00$
" " " " (c) = $\{1(625) + 1(500)\}/2 = 562.50$
" " " " (d) = $\{1(300) + 1(625) + 1(1000)\}/3 = 641.67$
" " " " (e) = $\{1(300) + 1(625) + 1(500)\}/3 = 475.00$

Before using these costs to determine the policy economy equation, we can caluculate the spending emphasis equation of implementing measure 2 as;

$$^{3}/8(a) + ^{2}/8(b) + ^{1}/8(c) + ^{1}/8(d) + ^{1}/8(e)$$

or in decimal form:

$$.375(a) + .250(b) + .125(c) + .125(d) + .125(e) \quad .$$

Since this policy involves using a single measures, the spending emphasis equation is the same for the cases of implementation cost available and unavailable.

To determine the equation describing the relative economy of the policy, we utilize our cost computations to calculate;

$$\frac{507.14}{625}(a) + \frac{675}{625}(b) + \frac{562}{625}(c) + \frac{641.67}{625}(d) + \frac{475}{625}(e) \quad ,$$

which normalizes to:

$$.1773(a) + .2500(b) + .1964(c) + .2243(d) + .1160(e) \quad .$$

Since the evaluating constituency in this example is equally sensitive to the spending emphasis in prevailing policy and the relative economy available from covering individual characteristics, the α parameter was determined to be 0.5. The composite equation would then be given as;

$$.25[(.375).1773(a) + (.250).2359(b) + (.125).1964(c) + .125(.2243)d + (.125).1160(e)]$$

This "composite" equation can then be used as a guide in allocating budget increases for augmenting the prevailing policy with additional measures. For example, suppose two new measures are proposed. Let measure 5 and measure 6 respectively be;

- A survey measuring the community's perception of the effectiveness of enforcement and the perceived detection and punishment liklihood. The implementation cost of this mail sampling procedure is estimated to be $2,800.
- A study of police and ambulance response times to reports of traffic accidents. The time spent by dispatching units for the necessary record keeping is estimated to cost $1,500.

The columns of the activity by measure matrix and the characteristic by measure matrix corresponding to these measures are;

Activity by Measure

measure ⑤	measure ⑥
1	0
0	0
1	0
0	1
1	0

Characteristic by Measure

measure ⑤	measure ⑥
2	0
2	0
0	1
1	1
1	0

For simplicity in this analysis, we will assume that all ISM elements are zero, and a budget increase for measure implementation of $8,000 has been awarded. Recall the costs of all available measure not currently being implemented are;

	Measure ①	③	④	⑤	⑥
Implementation Cost	1200	4000	2500	2800	1500

In light of the $8,000 budget increase, viable candidate measure sets (i.e., not subsets of another feasible set) which could be implemented to augment prevailing policy (measure 2) are the following:

Candidate Measure Combination		Total Implementation Cost
{ ① ③ ⑤ }	set C1	8000
{ ③ ④ ⑥ }	set C2	8000
{ ① ④ ⑤ ⑥ }	set C3	8000
{ ③ ① ⑥ }	set C4	6700
{ ① ③ ④ }	set C5	7700

Using the composite columns of the characteristic by measure matrix for each candidate set and the "composite" equation for determining augmentation consistent with prevailing policy, we calculate a "consistency score" for each candidate measure set as follows;

Candidate Combination	Score
C1→ .3345(4) + .2968(5) + .1237(0) + .1408(3) + .1046(2) =	3.453
C2→ .3345(3) + .2968(3) + .1237(2) + .1408(2) + .1046(1) =	2.528
C3→ .3345(5) + .2968(4) + .1237(2) + .1408(3) + .1046(3) =	3.843*
C4→ .3345(2) + .2968(3) + .1237(1) + .1408(3) + .1046(1) =	2.210
C5→ .3345(4) + .2968(4) + .1237(1) + .1408(2) + .1046(2) =	3.140

From the above, we can see that candidate measure set C3 has the highest consistency score with prevailing policy. As a result, the budget increase would be used to implement measures ①, ④, ⑤ and ⑥ if our objective were to remain consistent with prevailing measure policy, and achieve the best policy economy of characteristic coverage.

4.4 Designing a Substitute Measure

To see how the matrix methodology can be used as an aid in measure design, assume that the preferred measure for implementation is measure 4. If it were decided that this measure could not be implemented, and none of the other available measures were desirable, it would become necessary to design a new measure. Such a new measure would be designed to provide coverage within the characteristic by measure matrix which was at least surrogate to that provided by measure 4.

To determine the appropriate constraints we substitute the characteristic by activity matrix as the matrix of constraint coefficients. The right had side of the constraint set is given the column of the characteristic by measure matrix corresponding to measure 4, and, $\geq$, inequalities are inserted in the constraint expressions. The resulting constraint set is;

$$1X_1 + 1X_2 + 1X_3 + 0X_4 + 0X_5 \geq 2$$
$$1X_1 + 0X_2 + 1X_3 + 0X_4 + 0X_5 \geq 1$$
$$0X_1 + 1X_2 + 0X_3 + 1X_4 + 0X_5 \geq 1$$
$$1X_1 + 0X_2 + 0X_3 + 1X_4 + 0X_5 \geq 0$$
$$0X_1 + 0X_2 + 0X_3 + 1X_4 + 1X_5 \geq 1$$
$$X_i = 0 \text{ or } 1 \qquad i = 1...5$$

The solutions to this constraint set provide alternative 0-1 profiles within the activity by measure matrix which provide coverage which is at least surrogate to measure 4 within the characteristic by measure matrix. Figure 4 shows the branch and bound tree which enumerates all such alternatives. A total of four alternative profiles could be generate. They are:

$$\{0, 1, 1, 1, 1\}^t ,$$
$$\{1, 1, 1, 1, 1\}^t ,$$
$$\{1, 0, 1, 1, 1\}^t \text{ and}$$
$$\{1, 0, 1, 0, 1\}^t .$$

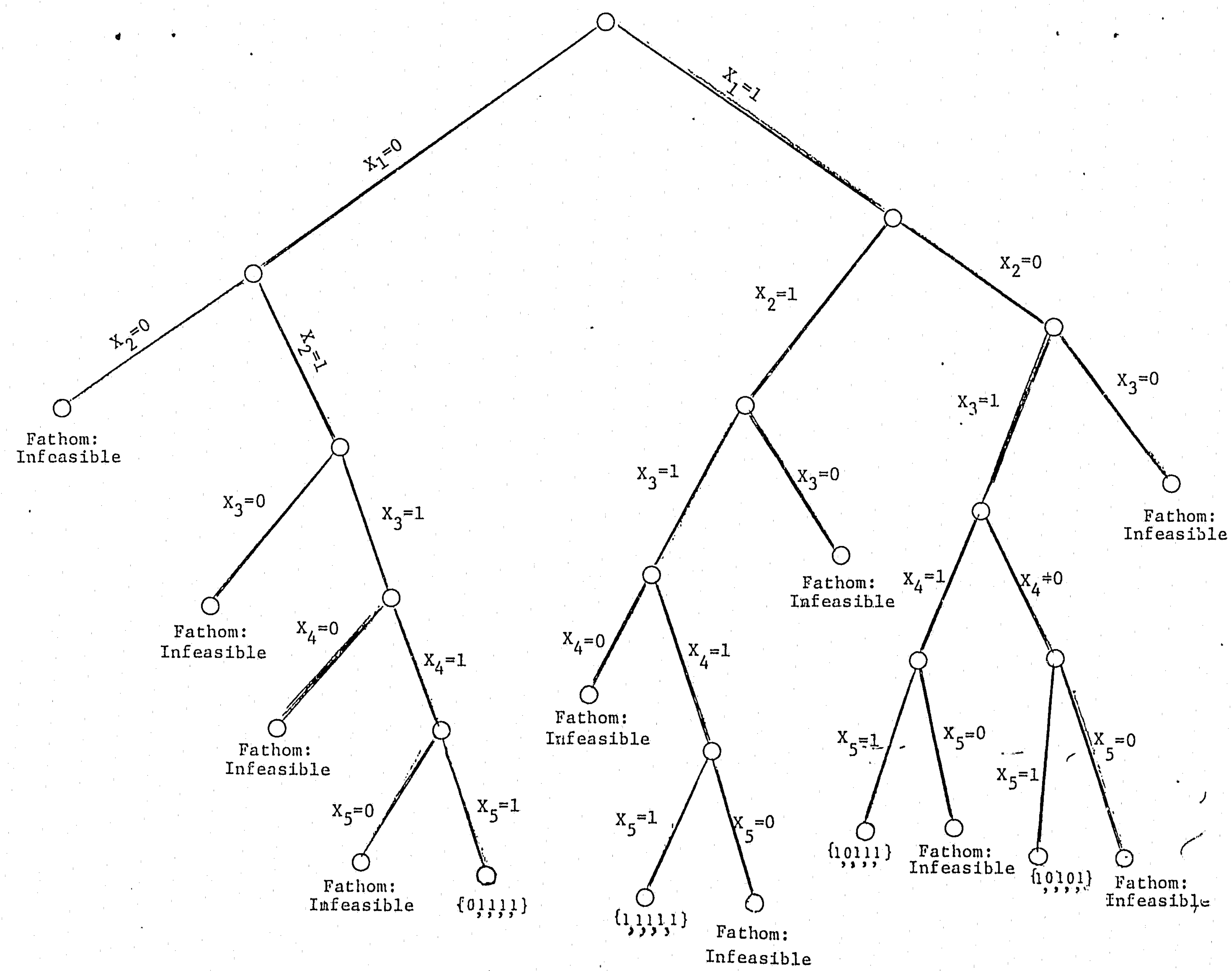

Figure [4]. The Branch and Bound Tree for the Design Problem

Using these profiles as a starting point, the next step of the measure design procedure would be to identify physical information sources giving rise to identical columns of the activity by measure matrix.

5. CONCLUSION

In this study we have developed a methodology for structuring performance measure problems within a matrix framework. It has been shown that this procedure can be used to develop quantitative indices which are useful for augmenting the information available to decision makers faced with an ill structured problem environment. The methodology is useful for selecting a set of measures for implementation from the set of all available measures, comparing measures and measure sets, analyzing the implications of measure policy, and designing new measures. Furthermore, implementing the method requires the evaluator to approach the problem in an orderly manner. Its generality makes the method applicable in nearly any context where model components can be identified.

Significant extensions to this research could include a methodology for evaluation of interactivity and intermeasure synergistic effects. Such a methodology could greatly enhance the usefulness of the matrix model in situations where strong interactions were though to exist between measures and other model components. In addition, the relationship between the determination of measure profiles and physical information sources could be clarified. This extension could enhance the ability of the matrix model to provide a tangible aid to measure design. The matrix model could also be embellished to find efficient ways for dealing with large scale performance measurement problems. This might

involve imbedding appropriate discrete optimization routines within the model and finding more effective ways of considering the properties of measure combinations.

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