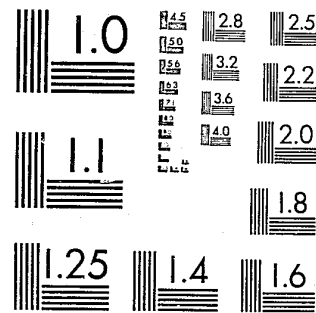


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Structural Covariance Models in Criminology:  
A Comparison of LISREL and PLS

by

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It is quite clear that causal modeling has the potential to make important contributions to understanding the origins of criminality. In particular, multivariate latent variable analysis, expressed in such forms as simultaneous equations, path analysis, and covariance structure analysis, will provide important insights into the causal process underlying criminal behavior. These extremely versatile analytic techniques require varying amounts of theoretical specification by a researcher; with sufficient specification and an appropriate data base, a theoretically meaningful model can be derived, tested, and revised iteratively. In the process, more exacting comparisons of two or more competing theoretical models of behavior can be made. As it is not possible to manipulate variables to evaluate causal influences in most criminal behavior research, and theory-testing is universally limited to non-experimental, correlational efforts, the newer multivariate methods provide powerful analytic tools.

By most standards, Jöreskog's (Jöreskog and Sörbom, 1978) full-information maximum-likelihood LISREL model represents the state-of-the-art in such modeling procedures. While the LISREL model has gained acceptance and adherents in varied quarters, e.g., Bentler (1980), Bielby and Hauser (1977), Kenny (1979), there are competing modeling procedures such as two-stage least-squares (2SLS), three-stage least-squares (3SLS), and partial least-squares (PLS). Methods such as ordinary least-squares (OLS), 2SLS, and 3SLS make some strong assumptions about the data (most notably, no measurement error) and occasionally require considerable ad hoc machinations in instances of overidentification, multicollinearity, etc., so as to obtain "best" parameter estimates. As few statistical tools are available for these least-squares methods, the researcher is forced to rely upon intuition, expertise, and vague criteria of goodness-of-fit.

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By contrast, the LISREL model offers some advantages over these least-squares competitors, e.g., allowing for multiple indicators of latent variables, assessment of measurement error, simultaneous correction for attenuation, and a full-information maximum-likelihood solution. But it may not provide a completely satisfactory alternative. In addition to non-trivial computational costs, LISREL exacts assumptions of multivariate normality and larger sample sizes. While statistical tests for goodness-of-fit and hypothesis-testing are available in LISREL, it has been argued that they can be grossly affected by sample size and violations of distributional assumptions (Bentler & Bonett, in press; McGarvey & Graham, Note 1; Olsson, 1979).

Wold's (1975) partial least-squares (PLS) has been presented as an alternative computational algorithm to LISREL. No distributional assumptions about manifest variables are required in PLS, and PLS can be used in single or multiple-indicator, recursive and non-recursive models (Jagpal & Hui, Note 2). As the PLS estimation procedure involves a rapid, iterative ordinary least-squares algorithm, substantially less computational time and cost has been found for PLS as compared to LISREL. A non-trivial problem with PLS, however, is its unstated, ill-defined, and/or undefined loss function. While some unpublished efforts have been directed to a comparison of PLS and LISREL, no thorough comparison of the two methodologies has been presented. Such a comparison is important for many reasons, not the least of which is the potential computational savings associated with the PLS algorithm.

The purposes of the present, limited investigation, then, are as follows. Using a plausible, substantively meaningful conceptual model, the performance characteristics of the LISREL and the PLS algorithms were examined. In addition to the substantive interpretations which the results of each analysis

will provide, such performance criteria include: (a) stability of parameter estimates; (b) computational time; (c) sensitivity of estimates to non-normally distributed data; and (d) residual distribution characteristics.

Substantive Model. Galliher and McCartney (1973) suggest that some of the variables most frequently studied as causes in delinquency research include: (a) social class of rearing environment; (b) educational attainment or school performance; (c) ethnicity; and (d) attitudes.

While often observed, the strength of the negative SES-criminality relationship has recently been questioned (Tittle, Villemez, and Smith, 1978). And, although the relationship of socioeconomic status to later criminality has a prominent position in theoretical positions such as Cohen's (1955), empirical studies of this covariation in the United States are subject to the confounded effects of ethnicity and social class. To compensate, some have suggested within-race analysis (Willie, 1967).

Another causal variable which historically has received varying degrees of attention in criminology is the concept of the "broken home" or "family status" (Wilkinson, 1974). While contradictory evidence has been obtained in testing the hypothesis, the home environment's impacts on juvenile delinquency form a critical part of theoretical statements such as those of Hirschi (1969).

Many elements can conceivably generate intra-familial stress. While the press of personal economic and emotional needs may make the family environment more or less cohesive and affect the social bonds between parents and children, social issues outside the family may also impact on these interpersonal relationships. In turn, these extra-familial social issues may affect the attachments, commitments, and beliefs (Hirschi, 1969) with which a child develops. Admittedly tenuous, this sort of "social strain" hypothesis may

provide a source of explanation for criminal behavior which is relatively orthogonal to the usually studied major influences.

The related concepts of academic aptitude and educational attainment have been received with varying levels of interest and antagonism in criminology (Hirschi and Hindelang, 1977). In many theories (e.g., Cloward & Ohlin, 1955; Cohen, 1955; Hirschi, 1969), the pursuit, receipt, and denial of academic rewards seem critical to the assimilation or rejection of middle-class virtues and standards.

To many criminologists, however, the concept of inherited characteristics (particularly intelligence) leading to scholastic achievement or failure is viewed as theoretically anathematic (Polk & Schafer, 1972). Objections to such "traditional" criminological attitudes have stirred some controversy (Hirschi & Hindelang, 1977) and the issues merit continued empirical and theoretical investigation.

In addition to these proposed causes of criminality, it might be that present, as opposed to rearing, social class is related to criminality. While this might be phrased in terms of lower SES leading to increased criminality either as a cathartic act, an equity restoration effort, or simply survival, it may also be that the consequences of criminality drastically limit social mobility. Alternatively, it might be argued that any covariation between present socioeconomic status and criminality is due to prior, common antecedents such as educational attainment, rearing social class, etc.

While concepts such as parental social class, being raised in a "broken home", "social strain", educational attainment, and current socioeconomic status do not exhaust the potential theoretical causes and correlates of criminality, there is every reason to believe that these constructs are of sufficient theoretical interest and import to merit further study. One poten-

tial structural model which might explain the inter-relationships of these constructs is depicted in Figure 1. In brief, the constructs

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 Insert Figure 1 about here  
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parental socio-economic status, "family status", and "social strain" are presumed to impact directly upon educational attainment. The impacts of parental socio-economic status, "family status," and "social strain" on criminality and respondent's socioeconomic status are, according to this model, mediated by educational attainment. While an unknown amount of the covariation between respondent's socioeconomic status and criminality is hypothesized as due to their common antecedents, some residual covariation is assumed to exist.

This description offers at least a skeletal operationalization of the model. While some subtle features require further explanation, we defer these issues until a later point in our discussion.

#### Method

Subjects. This study's samples were drawn from a true birth cohort, originally studied in an examination of criminality in XYY men (Witkin, Mednick, Schulsinger, Bakkestrom, Christiansen, Goodenough, Hirschhorn, Lundsteen, Owen, Philip, Rubin, & Stocking, 1976). This cohort consisted of all male children born during the interval 1 January, 1944 to 31 December, 1947 (N = 31,436), whose mothers were Copenhagen, Denmark residents.

As the purposes of the Witkin et al. (1976) study were substantively limited to the XYY chromosome issue, and Witkin et al. (1976) wished to maximize their chances of finding those with an XYY chromosomal pattern the most complete data (karyotyping, etc.) were compiled only for those equalling or exceeding 184 cm. in height. From these 4,578 taller subjects, complete

data for the present study could be found for 3,421 (75%). To effect a division in the sample for cross-validated tests, those subjects born on even-numbered days were assigned to sample A (N = 1,699 or 48%) while subjects born on odd-numbered days were assigned to sample B (1,772 or 52%).

Parental socio-economic status (JSESP). Using a scale based upon the efforts of Svalastoga (1959), the parental socioeconomic status was classified according to the father's occupation at the time of the respondent's birth (Psykologisk Institut Svalastoga Code).

Family status (LHOME). This dichotomous variate was used to indicate whether the respondent was raised in the standard nuclear family circumstances with both father and mother present (0 = No, 1 = Yes).

Post-war interval (IWTIM2). To assess the impact of the "social strain" of war on later criminality, the length of time between subjects' date of birth and the date of German surrender (7 May, 1945) was compiled in days. (Subjects born prior to 7 May, 1945 were coded zero). As this definition yielded an extreme skew, the data was subjected to a logarithmic transformation, viz,  $10 * (\log_{10} (X + 1))$ . The implication tested here is that those born either during or immediately after hostilities were subject to greater "social strain," impaired attachments, social bonds, etc., which would lead to greater impaired educational attainment and criminality in a later period.

Educational Test Index (IBPP1). As a measure of aptitude and scholastic achievement, the respondents' Borge Priens Prover scores (Rasch, 1960) were obtained from the Danish army draft board's records. This test has been shown to have a correlation of .70 with a Danish version of the WAIS (Moffitt, Gabrielli, Mednick, & Schulsinger, Note 3). Because the test was administered after the respondents had completed their schooling, and

because it is unclear to the authors to what extent this test measures either inherited aptitude or acquired knowledge, we feel safest in simply viewing it as a measure of educational performance.

Amount of School (SCHOOL). Derived from two different measures, this variate assessed the respondents' number of years of schooling in two different ways: (a) number of years of school if no academic examinations were passed; and (b) approximate equivalence of academic level of successfully passed examinations. Thus, if a respondent had only nine years of formal education, but passed an examination appropriate for twelve years of schooling, he was given a score of twelve on this measure.

Respondents' socioeconomic status. Three different measures of the respondents' socioeconomic status were gathered in the Witkin et al. (1976) study. One (ISES) was made on the subject as a young adult, prior to any personal interviews and attempted karyotyping. The other two measures (IHSES and IKSES) were asked as part of a later household interview. One (IHSES) corresponded exactly to the Psykologisk Institut Svalastoga Code in which the respondent's father was rated (JSESP). The other measure (IKSES), coded by K. O. Christiansen, was made according to Svalastoga's original scheme. The IKSES coding is not only in the reverse direction of all other SES measures used in this study, but it also is based upon different cutting points. (A high score on IKSES implies lower socioeconomic status).

Criminality. Six measures of criminality were selected for this study. A logarithmic transformation of the number of arrests, i.e.,  $10 * (\log_{10} (X + 1))$ , was computed (NARRST2). A scale of age at first arrest (FIRSTARR) was constructed after an examination of the frequency distribution for this variate. These ages and their respective codes are: (a) ages 0 - 10, 1; (b) ages 11 - 14, 2; (c) age 15, 3; (d) age 16, 4; (e) age 17, 5; (f) age 18, 6;

(g) age 19, 7; (h) age 20, 8; (i) age 21, 9; (j) age 22, 10; (k) age 23, 11; (l) ages 24 - 28, 12; (m) age 29 and never arrested, 13.

The other four measures of criminality in the present study were originally constructed by D. R. Owen in the Witkin, et al. (1976) study but were never used. For each of four sections of Danish penal code violations, Owens attempted to rank the severity of sanctions the violators received, either in terms of jail time or fines paid. The most severe of the sanctions which a respondent received for violations of the four sections of the Penal Code were recorded as the four sanction weights (SRWT1, SRWT2, SRWT3, SRWT4) which the present study will employ. Although K. O. Christiansen expressed doubts about the comparability of sanctions administered for similar offenses in different regions of Denmark, we feel the multiple measures we are using to represent the criminality construct can offset some of this problem. The manner in which these weights were derived, however, leads to another possible test of LISREL and PLS differences in parameter specification and estimation.

Measurement Model Specification. While Figure 1 depicts the relation of measured variables to latent variables (the measurement model), as well as the inter-relations of the unmeasured constructs (the structural model) a brief description of the measurement model seems in order.

The constructs -- parental social class, being raised in a "broken home" and "social strain" -- are represented here by the measures JSESP, LHOME and IWTIM2, respectively. The concept of "educational attainment" is operationalized as a factor on which the measures IBPP1 and SCHOOL have the only non-zero loadings. Respondent's socioeconomic status is measured by the three Svalastoga-Psykologisk Institut measures, IHSES, IKSES, and ISES. Respondent's criminality is viewed as the factor which underlies the measures

NARRST2, FIRSTARR, SRWT1, SRWT2, SRWT3, and SRWT4. Because the four sanction weights were derived in a similar manner, and because such a parameterization seemed an important way of testing possible LISREL/PLS differences, it seemed appropriate to permit correlated residuals among the sanction weights. Having the two additional indicators of the criminality construct will allow sufficient degrees of freedom to insure that minimally necessary identification conditions are met.

While such a measurement model is relatively straight-forwardly implemented in LISREL IV, it is not so easily transmuted into PLS terms. Boardman, Hui, and Wold (in press) claim -- erroneously -- that the LISREL program does not permit correlated errors, but that the PLS algorithm does. Furthermore, in Hui's (Note 4) PLSFP program documentation, it is unclear how such correlated residuals are to be specified for estimation. It should also be remarked that the residual covariation (or partial correlation) of the respondent's socioeconomic status and criminality constructs can not be very clearly parameterized in Hui's (Note 4) program. It remains, in short, unclear to the authors whether or not these terms are (a) estimable in Hui's (Note 4) PLSFP; (b) being estimated in Hui's (Note 4) PLSFP, but ignored; (c) absent in Hui's (Note 4) PLSFP computations. For present purposes, we will make assumptions (a) and (b).

#### Results

Descriptive Statistics. Summary statistical comparisons on each of the fourteen measures are presented in Table 1. Inspection of these data

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Insert Table 1 About Here  
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suggests two different conclusions: (a) none of the variates appear to be

normally distributed; and (b) sample A and sample B do not seem to differ greatly. By the Kolmogorov-Smirnov  $D$ -statistic, the probability of obtaining a  $D$ -statistic larger than that obtained for each variate is  $p < .01$ . It would thus be difficult to claim that these data in anyway meet univariate, much less multivariate normal distributional assumptions. Multivariate analysis of variance corroborates the conclusion of no sample differences, Wilks' criterion  $F(14,3406) = .77, p = .70$ . An exception to this is the variate SRWT3. Of the 3,421 males in samples A and B, only 13 had a non-zero SRWT3 score; 10 of these thirteen were assigned to sample A. Sample A, then, has a slightly higher mean severity of sanction here (.004 vs. .001; Duncan's test,  $p < .05$ ). Other statistics in Table 1, particularly the kurtosis and coefficient of variation, complement the conclusion that SRWT3 has a very peculiar distribution.

While we cannot, of course, logically establish the equivalence of Samples A and B in this manner, it is difficult to argue that they differ very much either.

Structural Model Analysis. As Jöreskog (1978) and others have argued, most appropriate uses of the chi-square goodness-of-fit test in confirmatory covariance structures analysis are model comparisons. To effect such a comparison, and to provide an alternative goodness-of-fit index, Bentler & Bonett (in press) have proposed a series of general model comparisons and a  $\Delta$  (delta) statistic. Table 2 summarizes the LISREL results for four such models. Note that no PLS statistics are provided because no such general models have been developed for PLS.

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 Insert Table 2 About Here  
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The first model listed for each sample is a so-called "null" model, which postulates no covariation whatsoever among the observed variates. The magnitude of these chi-squares suggests that complete independence is not a very tenable hypothesis. Next, a test of the hypothesis of uncorrelated factors (Sample A,  $\chi^2 = 2599.3174, 91 \text{ df.}, p < .0001, \Delta = .784$ ; Sample B,  $\chi^2 = 2517.1851, 91 \text{ df.}, p < .0001, \Delta = .792$ ) suggests two conclusions: (a) the variates chosen to represent the factors seem to be doing a reasonably good job; and (b) at least some of the factors are intercorrelated.

The next listings in Table 2 are the goodness-of-fit indices associated with the hypothesized model (Figure 1). While the chi-square tests in both samples suggest a rather poor fit, (Sample A,  $\chi^2 = 491.0784, 68 \text{ df.}, p < .0001$ ; Sample B,  $\chi^2 = 420.2346, 68 \text{ df.}, p < .0001$ ), the respective Bentler-Bonett  $\Delta$ s suggest that this conclusion may be more a function of sample size than quality of fit (Sample A,  $\Delta = .959$ ; Sample B,  $\Delta = .965$ ). That substantial improvements upon the hypothesized structural model are limited is illustrated by the last of the goodness-of-fit tests listed in Table 2. The model tested here postulates an oblique factor model, which permits the constructs to covary freely. While the chi-squares have dropped significantly in both samples, the  $\Delta$  coefficient in both samples has increased .004. To summarize, the hypothesized model may not be the only, nor necessarily even the best possible model for this data -- but it cannot be rejected as a completely unreasonable representation either.

Parameter Estimates. Tables 3 and 4 present the maximum-likelihood parameters, their standard errors, the standardized maximum-likelihood estimates, the partial least-squares estimates, the partial least-squares estimates' standard errors, the partial least-squares/two-stage least-squares parameter estimates, and their standard errors for samples A and B. (It should be



noted here that the maximum-likelihood or LISREL estimates for the  $\beta$  matrix are presented in accord with Bentler's (1980)  $R_{\lambda}$  notation; this makes the direction of their relations comparable to the  $\beta$  coefficients produced by PLS.)

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 Insert Tables 3 & 4 About Here  
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Substantive Interpretation. In addition to the previously stated conclusion about the adequacy of the measurement model, the coefficients in Tables 3 & 4 lead to the following substantive inferences: (a) parents' SES seems to exert a positive, significant influence on educational performance (e.g., in the sample A LISREL parameters,  $\beta_{4,1} = .446$ , S.E. = .022); (b) being raised in an environment with both parents present similarly has a positive, significant influence on educational performance (e.g.,  $\beta_{4,2} = .068$ ; S.E. = .022); (c) date of birth in relation to date of German surrender does not appear to be significantly related to educational attainment (e.g.,  $\beta_{4,3} = -.039$ , S.E. = .022); (d) later socioeconomic status is greatly influenced by educational attainment (e.g.,  $\beta_{5,4} = .812$ ; S.E. = .018); (e) lowered educational attainment is significantly associated with greater criminality (e.g.,  $\beta_{6,4} = -.399$ , S.E. = .024); (f) after controlling for parents' SES, being raised in a "broken home", "social strain", and educational attainment, the partial correlation between criminality and socioeconomic status is non-significant (in Sample A,  $\psi_{6,5} = -.001$ ; S.E. = .014; in Sample B,  $\psi_{6,5} = .006$ ; S.E. = .014).

Estimate Stability. Inspection of these coefficients reveals a high degree of comparability among the LISREL and PLS estimates both within samples and between samples. As a not altogether appropriate way of quantifying this impression, the comparable LISREL and PLS parameter estimates from Samples

A and B were correlated. The intercorrelations of these varying numbers of parameter estimates are given in Table 5.

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 Insert Table 5 About Here  
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Inspection of Table 5 reinforces the impression that these parameters are highly similar, both within and between samples.

But inspection of Tables 3 and 4 leads to at least three other conclusions. Firstly, despite slight differences in parameterization in the LISREL and PLS solutions, there are great many parameters that LISREL provides which PLS does not. Specifically, these are the measurement error terms,  $\theta_{\epsilon}$ . Additionally, the  $\psi_{4,4} - \psi_{6,6}$  terms in PLS were hand-computed, as PLS provides only estimated  $R^2$  terms for the endogenous constructs. Because PLS operates on the assumption of constructs having unit variance, the  $\psi_{4,4} - \psi_{6,6}$  terms were computed by subtracting the appropriate  $R^2$  from 1.0. Thirdly, the  $\lambda_{11,6} - \lambda_{14,6}$  maximum-likelihood estimates are uniformly depressed relative to the comparable partial least-squares parameter estimates. While not examined here, it should be noted that when no correlated residual terms were included in the LISREL model, these maximum-likelihood estimates were similarly inflated. Other than this discrepancy, the parameter estimates do appear to be very similar.

Computational Time. All computations were performed on an I.B.M. 3033 in an M.V.S. configuration. PLS computations were done in VS-APL, using Hui's (Note 4) PLSFP program. As expected, the partial least-squares algorithm derived its parameter estimates relatively rapidly, averaging approximately .8 seconds of C.P.U. time.



Even with this problem of relatively modest magnitude, however, the LISREL algorithm took much longer to derive a solution. With appropriate LISREL  $\Lambda$  elements fixed at 1.0 to set the latent variables' metrics, and all other free parameters starting at .1, LISREL required 4.87 seconds of C.P.U. time (initial loss function value = 43.2, final loss function value = .15). Instead of all possible output, requesting only minimal, default output cut LISREL's C.P.U. time in this case to 4.00 C.P.U. seconds.

To examine the effects of more optimal starting values, the maximum-likelihood parameter values obtained in Sample A were used as starting values in Sample B. This reduced C.P.U. time somewhat, to 3.45 C.P.U. seconds (initial loss function value = .18, final loss function value = .12). With the improved starting values, and only minimal output requested, LISREL's C.P.U. time in the Sample B solution was reduced to 2.53 C.P.U. seconds.

It appears, then, that PLS can derive its estimates much faster than LISREL, regardless of starting values or requested output; the magnitude of this difference should probably increase greatly with models of greater magnitude and/or complexity. Secondarily, and not unexpectedly, good starting values can speed up LISREL's estimation. To some degree, a comparison of PLSFP and LISREL C.P.U. times is not completely appropriate. As PLSFP is an APL program, and LISREL is a FORTRAN program, obviously different compilers are invoked for computations.

Sensitivity of Parameter Estimates to Non-Normality. As Tables 3, 4 and 5 all indicate, both the LISREL and PLS estimates appear rather stable despite the presence of clearly non-normal data. Exceptions to this stability are the maximum-likelihood and partial least-squares estimated factor loadings for the variate SRWT3 (viz.,  $\lambda_{13,6}$ ). These estimates differ from zero in the two PLS solutions and the Sample A LISREL solution. Note however that the

Sample B LISREL estimate of this parameter neither differs significantly from zero, nor shares overlapping confidence intervals with the Sample A estimates. It should be recalled here that this variate displayed a very peculiar univariate distribution, with an extremely large kurtosis relative to other variates in the model. It remains questionable as to whether this is the source of the observed differences. With this exception, the other parameter estimates appear to be relatively robust.

Residual Distribution Characteristics. Because the PLS loss function is so ill-defined relative to the LISREL or maximum-likelihood loss function, and because Hui's (Note 4) PLSFP program is apparently restricted to providing solutions only in correlational metric, the degree to which residual correlations for these algorithms and this model can be examined is rather limited. In addition, although Boardman, et. al. (in press) provide a formula for a reproduced  $\Sigma$ , or predicted covariance matrix, based upon the parameters which PLS computes, Hui's (Note 4) PLSFP program provides neither measurement error estimates nor a reproduced  $\Sigma$ . As a result, a correlational matrix of full rank cannot be reproduced from the obtained PLS estimates, without substituting a set of estimates based upon the assumption of unit variances in the observed variates. This makes either matrix-wise analysis of the reproduced correlations, or covariances, impossible in PLS; as a result, neither least-squares, generalized least-squares, nor maximum-likelihood criteria can be adequately tested with just the PLS estimates provided in Hui's (Note 4) routine.

Several steps were taken to compensate. First, the obtained PLS parameter estimates were substituted into the Boardman et. al. (in press) Formula 6; while there are some slight notational differences, this formula is very comparable to the reproduced  $\Sigma$  formula which Jöreskog & Sörbom (1978, p. 5, Formula 4) provide.

To denote their LISREL-like formulation, we label these reproduced  $\Sigma$ 's PLSL(A) and PLSL(B). Because we have derived an alternative reproduced  $\Sigma$  formula (see Appendix), the PLS estimates in Samples A and B were used to compute an additional pair of reproduced  $\Sigma$ 's. For convenience, we label these PLS (A) and PLS (B).

In addition to these four reproduced matrices, the analogous reproduced  $\Sigma$ 's from LISREL--labeled LISREL (A) and LISREL (B)--were also computed. The observed Sample A, Sample B, and the reproduced matrices are shown in Tables 6, 7, 8, and 9.

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 Insert Tables 6, 7, 8, and 9 about here  
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In a series of comparisons, the observed Sample A and Sample B correlation matrices were subtracted from these six reproduced matrices. For another comparative baseline, the Sample A correlation matrix was also subtracted from the Sample B correlation matrix. The lower triangular elements of each of the thirteen residual correlation matrices were written as a vector of 91 observations and subjected to descriptive statistical analysis. Summary statistics for these residual distributions' characteristics are presented in Table 10.

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 Insert Table 10 about here  
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Because none of the first moments of these distributions differed significantly from zero, first moments are not provided. Instead, the findings have been rank-ordered on the basis of the standard deviation of the residuals within validation and cross-validation samples. All residual distribu-

tions were roughly single-peaked, with little skew and varying degrees of kurtosis. Based upon these second moments' roots, it is of interest that the LISREL-reproduced coefficients, both in validation and cross-validation, seem to do a better job of fitting the observed correlations than either PLS formulation.

As a further, more appropriate test of the LISREL algorithm, the postulated structural model parameters were estimated in a covariance metric. As with the prior computations, the lower triangular elements of the five residual covariance matrices--plus the fourteen diagonal elements-- were written as 105 element vectors and subjected to similar descriptive statistical analysis.

The maximum-likelihood loss function was also computed for the covariance validation models and data, and for the covariance cross-validation models and data. As an additional index of fit, the Bentler-Bonett  $\Delta$  was also computed. Finally, a maximum-likelihood test for equality of covariance matrices (Morrison, 1976, p. 248, Eq. 3) was applied to the observed covariance matrices from Samples A and B. The summary statistics for the covariance comparisons are presented in Table 11.

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Turning to the descriptive statistics first, it should be noted that none of the first moments of the residuals differed significantly from zero. The distributions were again generally single-peaked, extremely leptokurtic, and centered on zero.

At least in cross-validated terms, Table 11 tends to deflate conclusions of adequacy of fit. While the  $\chi^2$  and  $\Delta$ 's tests in the validation samples are very high, and concur with those obtained in the correlational analysis (see

Table 2), the chi-squares and  $\Delta$ 's computed in cross-validation have shrunken rather drastically. It is of interest to note, however, that the proposed structural model for Sample A predicts the Sample B covariance matrix ( $\Delta = .838$ ) somewhat less well than the observed Sample A covariance matrix predicts the sample B covariance matrix ( $\Delta = .865$ ). Similarly, the proposed structural model for Sample B predicts the Sample A covariance matrix ( $\Delta = .625$ ) somewhat less well than does the observed B covariance matrix ( $\Delta = .661$ ).

#### DISCUSSION

Substantive Model Implications. The results of these several tests support the conceptual viability of the proposed structural model in most regards, with two notable exceptions. The concept of "social strain" may have some intuitive appeal, but at least as operationalized here, it does not seem to contribute very much to educational attainment, criminality, or later socioeconomic status. Secondly, it is of some interest that the partial correlation between criminality and socioeconomic status ( $\psi_{6,5}$ ) is negligible. This argues very strongly for determination of, and generating policy impacts upon, common antecedents such as educational attainment.

Parental socioeconomic status does seem to play a major contributing role in the determination of educational attainment, and thus indirectly upon later socioeconomic status and criminality. Being raised in a "broken home" seems to contribute relatively less to these outcomes.

Finally, while it may be unique either to the culture or the operationalizations we have chosen, the very high proportion of variance accounted for in respondent socioeconomic status (approximately 70%) is somewhat remarkable. Comparable examinations in standard social mobility models in a recent U. S.-Mexico comparison were in the neighborhood of 20% to 30% (McGarvey,

Fairbanks, and Sukoneck, Note 5). The reasons for such relatively good performance remain unclear.

By contrast, the model seems to account for some 12% to 15% of the variance in criminality. While this is certainly limited relative to the status attainment component described above, it is not altogether unrealistic for similar sorts of structural models in criminology.

Methodological Criteria. Despite patently obvious violations of multivariate normality distributional assumptions, both LISREL and PLS seem to provide reasonable, replicable results. PLS seems to provide most of the important parameter estimates in considerably less computational time, but with some loss in both capabilities of complex parameterization and in terms of fitting the observed correlation matrix. How critical these absent parameters, less complete parameterization, and increased error components are, relative to computational cost advantages, will depend upon the particular structural model under consideration and user preferences.

An area for further examination would involve developing a PLS-LISREL interface. In this manner, the PLS estimates could be used as relatively optimal starting values for the LISREL solution. Under a VSAPL computational configuration, such an interface could be established in fairly straightforward terms.

The adequacy of PLS in certain cases where LISREL shows clear advantages, such as parameter constraints, multiple-group analysis, and covariance metric analysis, is yet to be developed. Such developments, in conjunction with a PLS-LISREL interface, could permit expanded applications of both techniques.

The LISREL algorithm, it seems, suffers most clearly in comparison to PLS in computational costs. If some option were available to the LISREL user

to permit a "rougner", less precise solution, this would help--particularly in more exploratory applications. Presently, the only relevant option is to set the C.P.U. request at a minimal level; what we are proposing is that stopping criteria, or changes in the loss function be under more direct user control. This will prevent several iterations of the algorithm with little or no loss function change.

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Table 1  
Univariate Descriptive Statistics for LISREL - PLSFP Comparison

Sample	Minima		Maxima		Mean		Variance		Skewness		Kurtosis		Coefficient of Variation	
	A	B	A	B	A	B	A	B	A	B	A	B	A	B
<u>Variate</u>														
JSESP	1	1	7	7	3.61	3.66	2.89	2.80	.2	.2	-.6	-.6	47.2	45.8
LHOME	0	0	1	1	.89	.88	.10	.11	-2.4	-2.3	3.8	3.5	36.1	37.0
IWTIM2	0	0	29.8	29.8	16.42	16.77	158.51	160.24	-.4	-.5	-1.7	-1.6	76.7	75.5
IBPP1	1	1	9	9	5.49	5.49	4.19	4.04	-.1	-.1	-.6	-.6	37.3	36.7
SCHOOL	5	5	12	12	10.16	10.17	2.93	2.82	-.7	-.7	-.8	-.8	16.9	16.5
IHSES	1	1	7	7	4.49	4.51	3.10	2.95	.1	.1	-1.0	-.9	39.2	38.1
IKSES	3	3	9	9	6.11	6.08	1.09	1.09	-.5	-.5	-.0	-.2	17.1	17.2
ISES	1	1	7	7	4.26	4.25	2.92	2.75	.3	.3	-.8	-.7	40.1	39.0
NARRST2	0	0	15.9	13.2	1.34	1.27	5.83	5.38	2.1	2.0	4.8	3.7	180.0	183.1
FIRSTARR	1	1	13	13	11.56	11.62	7.85	7.44	-2.0	-2.0	3.0	3.1	24.2	23.5
SRWT1	0	0	.9	.9	.06	.05	.04	.03	3.3	3.4	9.2	10.4	330.8	338.8
SRWT2	0	0	.8	.8	.02	.02	.01	.01	6.3	6.4	39.0	40.5	633.8	642.9
SRWT3	0	0	.9	.6	.00	.00	.00	.00	14.0	24.6	198.3	611.4	1340.2	2442.5
SRWT4	0	0	.9	.9	.04	.03	.02	.02	3.8	4.2	13.1	16.9	389.5	429.4

Table 2  
Fit Indices for Hypothesized Covariance Structures

Model	Sample A				Sample B			
	$\chi^2$	df	p	$\Delta$	$\chi^2$	df	p	$\Delta$
Null	12042.74	91	<.001	-	12088.60	91	<.001	-
Orthogonal	2599.32	74	<.001	.78	2517.19	74	<.001	.74
Hypothesized	491.08	68	<.001	.96	420.23	68	<.001	.97
Oblique	449.11	59	<.001	.96	375.88	59	<.001	.97

Table 3  
LISREL and PLSFP Estimates, Sample A

PARAMETER	M.L.	M.L.S.E	ST.M.L.	PLS	PLS S.E.	PLS 2SLS	PLS 2SLS S.E.
$\lambda_{1,1}$	1.000*	0.000	1.000	1.000	0.000	.	.
$\lambda_{2,2}$	1.000*	0.000	1.000	1.000	0.000	.	.
$\lambda_{3,3}$	1.000*	0.000	1.000	1.000	0.000	.	.
$\lambda_{4,4}$	1.000*	0.000	0.972	0.964	0.007	.	.
$\lambda_{5,4}$	0.884	0.017	0.860	0.952	0.008	.	.
$\lambda_{6,5}$	1.000	0.000	0.945	0.955	0.007	.	.
$\lambda_{7,5}$	-0.836	0.019	-0.790	-0.861	0.013	.	.
$\lambda_{8,5}$	0.915	0.017	0.864	0.905	0.011	.	.
$\lambda_{9,6}$	1.000	0.000	0.991	0.924	0.009	.	.
$\lambda_{10,6}$	-0.871	0.018	-0.864	-0.865	0.012	.	.
$\lambda_{11,6}$	0.661	0.021	0.656	0.820	0.014	.	.
$\lambda_{12,6}$	0.345	0.024	0.342	0.452	0.022	.	.
$\lambda_{13,6}$	0.288	0.024	0.286	0.381	0.023	.	.
$\lambda_{14,6}$	0.491	0.023	0.487	0.655	0.019	.	.
$\beta_{4,1}$	0.446	0.022	0.459	0.446	0.022	0.446	0.022
$\beta_{4,2}$	0.068	0.022	0.070	0.061	0.022	0.061	0.022
$\beta_{4,3}$	-0.039	0.022	-0.040	-0.034	0.022	-0.034	0.022
$\beta_{5,4}$	0.812	0.018	0.836	0.777	0.016	0.883	0.050
$\beta_{6,4}$	-0.399	0.024	-0.391	-0.369	0.023	-0.379	0.054
$\psi_{1,1}$	1.000	0.035	1.000	1.000	.	1.000	.
$\psi_{2,2}$	1.000	0.035	1.000	1.000	.	1.000	.
$\psi_{3,3}$	1.000	0.035	1.000	1.000	.	1.000	.
$\psi_{4,4}$	0.740	0.029	0.783	0.796	.	0.796	.



Table 3 (cont'd.)  
LISREL and PLSFP Estimates, Sample A

PARAMETER	M.L.	M.L.S.E.	ST. M.L.	PLS	PLS S.E.	PLS 2SLS	PLS 2SLS S.E.
$\psi_{5,5}$	0.269	0.015	0.301	0.396	.	0.841	.
$\psi_{6,6}$	0.833	0.033	0.847	0.864	.	0.971	.
$\psi_{6,5}$	-0.001	0.014	-0.001	.	.	.	.
$\theta_{\varepsilon_{1,1}}$	0.000*	0.000	.	.	.	.	.
$\theta_{\varepsilon_{2,2}}$	0.000*	0.000	.	.	.	.	.
$\theta_{\varepsilon_{3,3}}$	0.000*	0.000	.	.	.	.	.
$\theta_{\varepsilon_{4,4}}$	0.055	0.010	.	.	.	.	.
$\theta_{\varepsilon_{5,5}}$	0.261	0.012	.	.	.	.	.
$\theta_{\varepsilon_{6,6}}$	0.107	0.009	.	.	.	.	.
$\theta_{\varepsilon_{7,7}}$	0.376	0.015	.	.	.	.	.
$\theta_{\varepsilon_{8,8}}$	0.254	0.012	.	.	.	.	.
$\theta_{\varepsilon_{9,9}}$	0.017	0.014	.	.	.	.	.
$\theta_{\varepsilon_{10,10}}$	0.254	0.014	.	.	.	.	.
$\theta_{\varepsilon_{11,11}}$	0.570	0.021	.	.	.	.	.
$\theta_{\varepsilon_{12,12}}$	0.883	0.031	.	.	.	.	.
$\theta_{\varepsilon_{13,13}}$	0.918	0.032	.	.	.	.	.
$\theta_{\varepsilon_{14,14}}$	0.763	0.027	.	.	.	.	.
$\theta_{\varepsilon_{12,11}}$	0.077	0.018	.	.	.	.	.
$\theta_{\varepsilon_{13,11}}$	0.110	0.018	.	.	.	.	.
$\theta_{\varepsilon_{14,11}}$	0.192	0.018	.	.	.	.	.
$\theta_{\varepsilon_{13,12}}$	0.085	0.022	.	.	.	.	.
$\theta_{\varepsilon_{14,12}}$	0.162	0.021	.	.	.	.	.
$\theta_{\varepsilon_{14,13}}$	0.061	0.021	.	.	.	.	.

\*Coefficient was fixed at this value during estimation.

Table 4  
LISREL and PLSFP Estimates, Sample B

PARAMETER	M.L.	M.L.S.E.	ST.M.L.	PLS	PLS S.E.	PLS 2SLS	PLS 2SLS S.E.
$\lambda_{1,1}$	1.000*	0.000	1.000	1.000	0.000	.	.
$\lambda_{2,2}$	1.000*	0.000	1.000	1.000	0.000	.	.
$\lambda_{3,3}$	1.000*	0.000	1.000	1.000	0.000	.	.
$\lambda_{4,4}$	1.000*	0.000	0.956	0.958	0.007	.	.
$\lambda_{5,4}$	0.888	0.018	0.849	0.945	0.008	.	.
$\lambda_{6,5}$	1.000	0.000	0.949	0.954	0.007	.	.
$\lambda_{7,5}$	-0.827	0.018	-0.785	-0.859	0.012	.	.
$\lambda_{8,5}$	0.910	0.016	0.863	0.907	0.010	.	.
$\lambda_{9,6}$	1.000	0.000	0.967	0.914	0.010	.	.
$\lambda_{10,6}$	-0.916	0.018	-0.885	-0.875	0.012	.	.
$\lambda_{11,6}$	0.655	0.021	0.633	0.788	0.015	.	.
$\lambda_{12,6}$	0.393	0.024	0.380	0.525	0.020	.	.
$\lambda_{13,6}$	0.058	0.025	0.056	0.135	0.024	.	.
$\lambda_{14,6}$	0.475	0.023	0.459	0.646	0.018	.	.
$\beta_{4,1}$	0.406	0.021	0.424	0.403	0.022	0.403	0.022
$\beta_{4,2}$	0.085	0.021	0.089	0.086	0.022	0.086	0.022
$\beta_{4,3}$	-0.018	0.021	-0.019	-0.013	0.022	-0.013	0.022
$\beta_{5,4}$	0.830	0.019	0.836	0.770	0.015	0.922	0.053
$\beta_{6,4}$	-0.357	0.024	-0.353	-0.326	0.023	-0.317	0.057
$\psi_{1,1}$	1.000	0.034	1.000	1.000	.	1.000	.
$\psi_{2,2}$	1.000	0.034	1.000	1.000	.	1.000	.
$\psi_{3,3}$	1.000	0.034	1.000	1.000	.	1.000	.
$\psi_{4,4}$	0.743	0.029	0.812	0.830	.	0.830	.

Table 4 (cont'd.)  
LISREL and PLSFP Estimates, Sample B

PARAMETER	M.L.	M.L.S.E.	ST. M.L.	PLS	PLS S.E.	PLS 2SLS	PLS 2SLS S.E.
$\psi_{5,5}$	0.271	0.015	0.301	0.408		0.856	
$\psi_{6,6}$	0.819	0.032	0.875	0.894		0.983	
$\psi_{6,5}$	0.008	0.014	0.008				
$\theta_{\epsilon_{1,1}}$	0.000*	0.000					
$\theta_{\epsilon_{2,2}}$	0.000*	0.000					
$\theta_{\epsilon_{3,3}}$	0.000*	0.000					
$\theta_{\epsilon_{4,4}}$	0.086	0.011					
$\theta_{\epsilon_{5,5}}$	0.280	0.013					
$\theta_{\epsilon_{6,6}}$	0.099	0.009					
$\theta_{\epsilon_{7,7}}$	0.385	0.015					
$\theta_{\epsilon_{8,8}}$	0.255	0.011					
$\theta_{\epsilon_{9,9}}$	0.065	0.014					
$\theta_{\epsilon_{10,10}}$	0.216	0.013					
$\theta_{\epsilon_{11,11}}$	0.599	0.021					
$\theta_{\epsilon_{12,12}}$	0.855	0.029					
$\theta_{\epsilon_{13,13}}$	0.997	0.034					
$\theta_{\epsilon_{14,14}}$	0.789	0.027					
$\theta_{\epsilon_{12,11}}$	0.100	0.018					
$\theta_{\epsilon_{13,11}}$	0.091	0.019					
$\theta_{\epsilon_{14,11}}$	0.148	0.017					
$\theta_{\epsilon_{13,12}}$	0.040	0.022					
$\theta_{\epsilon_{14,12}}$	0.204	0.020					
$\theta_{\epsilon_{14,13}}$	0.088	0.021					

\*Coefficient was fixed at this value during estimation.

Table 5  
Intercorrelations of LISREL and PLSFP Estimates,  
Samples A and B

M.L.(A)	1.000							
ST.M.L.(A)	.999	1.000						
PLS(A)	.994	.994	1.000					
PLS2SLS(A)	.940	.946	.962	1.000				
M.L.(B)	.996	.995	.988	.938	1.000			
ST.M.L.(B)	.995	.996	.988	.944	.999	1.000		
PLS(B)	.991	.991	.995	.962	.993	.993	1.000	
PLS2SLS(B)	.936	.942	.959	.999	.935	.941	.960	1.000

Table 6  
PLSL(A) and PLSL(B) Reproduced Matrices

JSESP	-	.000	.000	.386	.381	.296	-.267	.281	-.120	.115	-.104	-.069	-.018	-.085
LHOME	.000	-	.000	.082	.081	.063	-.057	.060	-.026	.025	-.022	-.015	-.004	-.018
IWTIM2	.000	.000	-	-.012	-.012	-.010	.009	-.009	.004	-.004	.003	.002	.001	.003
IBPP1	.430	.059	-.033	-	.905	.704	-.634	.669	-.285	.273	-.246	-.164	-.042	-.202
SCHOOL	.425	.058	-.032	.918	-	.694	-.625	.660	-.282	.270	-.243	-.162	-.042	-.199
IHSES	.331	.045	-.025	.715	.706	-	-.820	.866	-.219	.210	-.189	-.126	-.032	-.155
IKSES	-.298	-.041	.023	-.645	-.637	-.822	-	-.780	.197	-.189	.170	.113	.029	.139
ISES	.314	.043	-.024	.678	.669	.864	-.779	-	-.208	.199	-.179	-.120	-.031	-.147
NARRST2	-.152	-.021	.012	-.329	-.325	-.253	.228	-.240	-	-.800	.720	.480	.123	.591
FIRSTARR	.142	.019	-.011	.308	.304	.237	-.213	.224	-.799	-	-.690	-.460	-.118	-.565
SRWT1	-.135	-.018	.010	-.292	-.288	-.224	.202	-.213	.758	-.709	-	.414	.106	.509
SRWT2	-.074	-.010	.006	-.161	-.159	-.124	.112	-.117	.418	-.391	.371	-	.071	.339
SRWT3	-.063	-.009	.005	-.135	-.134	-.104	.094	-.099	.352	-.330	.312	.172	-	.087
SRWT4	-.108	-.015	.008	-.233	-.230	-.179	.162	-.170	.605	-.567	.537	.296	.250	-
DIAGONAL ELEMENTS														
Sample A	1.000	1.000	1.000	.929	.906	.912	.741	.819	.854	.748	.672	.204	.145	.429
Sample B	1.000	1.000	1.000	.918	.893	.911	.739	.823	.836	.766	.621	.276	.018	.417

Note: The reproduced correlations from Sample A are provided in the lower triangle, while the reproduced correlations from Sample B are provided in the upper triangle.

Table 7  
PLS(A) and PLS(B) Reproduced Matrices

JSESP	-	-.007	.004	.398	.367	.352	-.279	.371	-.114	.130	-.114	-.026	-.018	-.069
LHOME	.038	-	.047	.085	.072	.064	-.071	.074	-.014	.042	-.017	-.019	.028	-.010
IWTIM2	.064	.023	-	-.002	-.014	.015	.065	.018	-.008	-.016	-.013	.011	.009	.008
IBPP1	.443	.074	-.001	-	1.195	.857	-.674	.885	-.377	.364	-.285	-.149	-.046	-.228
SCHOOL	.411	.074	-.008	1.252	-	.805	-.619	.840	-.339	.327	-.247	-.126	-.040	-.200
IHSES	.377	.066	.017	.862	.809	-	-.725	.829	-.246	.232	-.198	-.094	-.030	-.148
IKSES	-.293	-.063	.054	-.691	-.636	-.729	-	-.700	.195	-.185	.161	.078	.025	.119
ISES	.393	.057	.018	.909	.869	.829	-.707	-	-.269	.256	-.216	-.106	-.033	-.163
NARRST2	-.153	-.032	-.009	-.436	-.397	-.275	.226	-.288	-	-.805	.719	.477	.127	.592
FIRSTARR	.174	.049	-.028	.416	.381	.251	-.207	.266	-.794	-	-.688	-.457	-.121	-.566
SRWT1	-.154	.010	.003	-.357	-.318	-.230	.191	-.240	.750	-.703	-	.410	.109	.507
SRWT2	-.022	-.016	.021	-.138	-.120	-.070	.059	-.075	.411	-.385	.365	-	.072	.338
SRWT3	-.045	.024	.004	-.115	-.096	-.072	.062	-.074	.346	-.323	.310	.172	-	.090
SRWT4	-.088	-.037	-.042	-.235	-.203	-.151	.125	-.156	.591	-.552	.525	.290	.245	-
DIAGONAL ELEMENTS														
Sample A	1.000	1.000	1.000	1.240	1.284	.843	.645	.826	.847	.748	.668	.206	.147	.419
Sample B	1.000	1.000	1.000	1.194	1.213	.847	.638	.818	.841	.772	.618	.276	.020	.418

Note: The reproduced correlations from Sample A are provided in the lower triangle, while the reproduced correlations from Sample B are provided in the upper triangle.

Table 8  
LISREL(A) and LISREL(B) Reproduced Matrices

JSESP	-	.000	.000	.406	.360	.337	-.278	.306	-.145	.133	-.095	-.057	-.008	-.069
LHOME	.000	-	.000	.085	.075	.070	-.058	.064	-.030	.028	-.020	.012	-.002	-.014
IWTIM2	.000	.000	-	-.018	-.016	-.015	.012	-.014	.006	-.006	.004	.003	.000	.003
IBPP1	.446	.068	-.039	-	.812	.759	-.627	.691	-.326	.299	-.214	-.128	-.019	-.155
SCHOOL	.395	.060	-.035	.836	-	.674	-.557	.613	-.290	.265	-.190	-.114	-.017	-.138
IHSES	.362	.055	-.032	.768	.679	-	-.745	.820	-.263	.241	-.172	-.104	-.015	-.125
IKSES	-.303	-.046	.027	-.642	-.568	-.746	-	-.677	.218	-.199	.142	.086	.013	.103
ISES	.332	.050	-.029	.702	.621	.816	-.683	-	-.239	.219	-.157	-.094	-.014	-.114
NARRST2	-.178	-.027	.016	-.377	-.333	-.307	.257	-.281	-	-.856	.612	.368	.054	.444
FIRSTARR	.155	.024	-.014	.328	.290	.267	-.224	.244	-.856	-	-.561	-.337	-.049	-.407
SRWT1	-.118	-.018	.010	-.249	-.220	-.203	.170	-.186	.650	-.566	-	.341	.126	.438
SRWT2	-.061	-.009	.005	-.130	-.115	-.106	.089	-.097	.339	-.295	.301	-	.062	.379
SRWT3	-.051	-.008	.005	-.109	-.096	-.088	.074	-.081	.283	-.247	.298	.183	-	-.113
SRWT4	-.087	-.013	.008	-.185	-.164	-.151	.126	-.138	.483	-.421	.512	.329	.201	-

Note: The reproduced correlations from Sample A are provided in the lower triangle, while the reproduced correlations from Sample B are provided in the upper triangle.

Table 9  
Observed Correlation Matrices, Samples A and B

JSESP	-	-.007	.004	.398	.367	.357	-.284	.375	-.096	.113	-.099	-.016	-.016	-.056
LHOME	.038	-	.047	.084	.074	.031	-.038	.049	-.101	.126	-.091	-.070	.015	-.072
IWTIM2	.064	.023	-	-.006	-.009	-.040	.122	-.028	-.041	.017	-.039	-.008	.004	-.016
IBPP1	.442	.080	-.006	-	.812	.739	-.590	.767	-.324	.313	-.255	-.136	-.052	-.199
SCHOOL	.412	.067	-.002	.836	-	.657	-.531	.658	-.273	.266	-.209	-.104	-.020	-.167
IHSES	.369	.094	-.032	.745	.660	-	-.774	.814	-.251	.229	-.218	-.105	-.036	-.152
IKSES	-.285	-.092	.104	-.615	-.537	-.780	-	-.617	.204	-.198	.181	.100	.022	.130
ISES	.387	.079	-.022	.779	.672	.811	-.615	-	-.265	.250	-.225	-.107	-.040	-.174
NARRST2	-.141	-.121	-.011	-.378	-.313	-.296	.251	-.307	-	-.857	.608	.379	.058	.451
FIRSTARR	.163	.131	-.024	.359	.301	.265	-.236	.271	-.856	-	-.572	-.295	-.033	-.376
SRWT1	-.143	-.071	.002	-.318	-.259	-.268	.234	-.242	.649	-.578	-	.341	.126	.438
SRWT2	-.016	-.061	.017	-.123	-.095	-.085	.075	-.085	.342	-.245	.301	-	.062	.379
SRWT3	-.040	-.016	.004	-.111	-.082	-.086	.085	-.093	.287	-.187	.298	.183	-	.113
SRWT4	-.079	-.103	-.043	-.202	-.186	-.181	.167	-.168	.485	-.383	.512	.329	.201	-

Note: The correlations from Sample A are provided in the lower triangle, while the correlations from Sample B are provided in the upper triangle.

Table 10  
Univariate Statistics, Residual Correlations

<u>Predicted</u>	<u>Observed</u>	<u>Minima</u>	<u>Maxima</u>	<u>Standard Deviation</u>	<u>Skewness</u>	<u>Kurtosis</u>
LISREL(B)	S <sub>B</sub>	-.110	.071	.034	-.527	1.340
LISREL(A)	S <sub>A</sub>	-.107	.094	.035	.124	.865
PLSL(B)	S <sub>B</sub>	-.189	.139	.057	-.634	1.474
PLSL(A)	S <sub>A</sub>	-.183	.120	.058	-.456	.818
PLS(B)	S <sub>B</sub>	-.190	.383	.071	1.502	8.458
PLS(A)	S <sub>A</sub>	-.169	.416	.073	1.837	10.208
S <sub>A</sub>	S <sub>B</sub>	-.154	.229	.050	1.154	5.106
LISREL(A)	S <sub>B</sub>	-.214	.225	.056	.453	4.463
LISREL(B)	S <sub>A</sub>	-.233	.138	.061	-.637	1.487
PLSL(B)	S <sub>A</sub>	-.214	.138	.072	-.586	.242
PLS(B)	S <sub>A</sub>	-.212	.359	.079	.536	4.451
PLSL(A)	S <sub>B</sub>	-.296	.295	.080	.172	3.340
PLS(A)	S <sub>B</sub>	-.290	.440	.095	.947	4.752

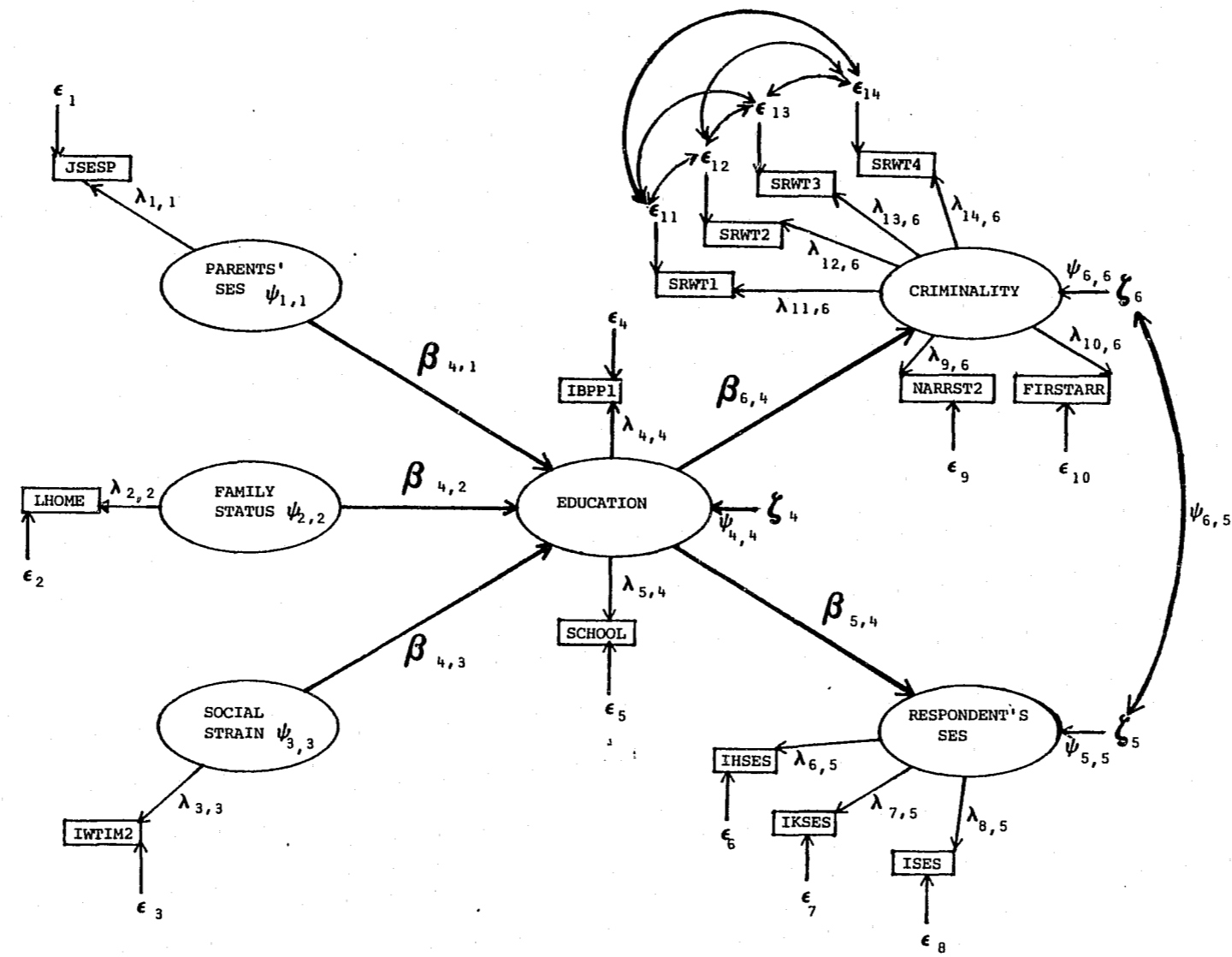


Table 11  
Univariate Statistics and Fit Indices, Residual Covariances

<u>Predicted</u>	<u>Observed</u>	<u>Minima</u>	<u>Maxima</u>	<u>Standard Deviation</u>	<u>Skewness</u>	<u>Kurtosis</u>	<u><math>\chi^2</math></u>	<u><math>\Delta</math></u>
LISREL(A)	S <sub>A</sub>	-1.372	.818	.222	-3.108	19.213	491.088	.959
LISREL(B)	S <sub>B</sub>	-1.450	1.406	.230	-.359	29.001	420.301	.965
S <sub>A</sub>	S <sub>B</sub>	-1.742	1.298	.298	-2.102	19.481	1627.27	.865
S <sub>B</sub>	S <sub>A</sub>	-	-	-	-	-	4078.70	.661
LISREL(B)	S <sub>A</sub>	-1.372	1.742	.297	.643	17.110	4510.51	.625
LISREL(A)	S <sub>B</sub>	-1.742	1.692	.328	-.996	15.898	1953.32	.838

Figure 1

Path Model for Relations Among Parents' SES, Family Status, Social Strain,  
Education, Respondent's SES and Criminality.



## Appendix I

Obtaining Estimates of Measured Variable Correlations

PLS is essentially a method for estimating the parameters of the measurement model (i.e., "outer relations") portion of a latent variable causal model, while taking into account the underlying structural relationships among causally related latent variables. Once these "outer relations" have been estimated, any of a number of structural equation estimation techniques may be used to obtain parameter estimates of the structural relationships among latent variables.

Unfortunately the PLS implementation used for our analyses did not provide estimates of the measured and residual variable correlation matrices, which are typically very useful in assessing the overall goodness of fit of a given structural model to empirical data. This appendix will briefly present the necessary equations for generating such estimates.

Estimated Correlations From the Measurement Model

$$(1) A'R_{yy}A = COV_{LL}$$

$$(2) D = DIAG(COV_{LL})^{-\frac{1}{2}}$$

$$(3) DA'R_{yy}AD = R_{LL}$$

$$(4) R_{yy}AD = F_s$$

$$(5) F_p = F_s R_{LL}^{-1}$$

$$(6a) \hat{R}_{yy} = F_p R_{LL} F_p'$$

$$(6b) \hat{R}_{yy} = F_s R_{LL}^{-1} R_{LL} R_{LL}^{-1} F_s' = F_s R_{LL}^{-1} F_s'$$

$$(7) R_{EE} = R_{yy} - \hat{R}_{yy}$$

Definition of Symbols

$R_{yy}$  - is a correlation matrix among the m measured variables (m×m) (MV's) of the model.

A - is a weighting matrix for generating a set of k latent variables (LV's) as linear composites of the MV's. (m×k)

$COV_{LL}$  - is a covariance matrix among the k LV's of the model. (k×k)

D - is a diagonal scaling matrix used to standardize the k LV's to unit variance. (k×k)

$R_{LL}$  - is the latent variable correlation matrix. (k×k)

$F_s$  - is the cross-correlation matrix between MV's and LV's. (m×k)  
In factor analytic terms, it is known as the factor structure matrix.

$F_p$  - is the path coefficient matrix of regression weights for estimating each MV from its respective LV. In factor analytic terms, it is known as the factor pattern matrix. (m×k)

$\hat{R}_{yy}$  - is the estimated MV correlation matrix generated from estimates of the model's outer structural relationships. (m×m)

$R_{EE}$  - is the residual correlation matrix. (m×m)

### The Generating Equations

The unmodified PLS algorithm provides final estimates of the elements of the weighting matrix A for generating LV's as linear composites of the MV's. Given this weighting matrix, the LV covariance matrix may be generated as indicated in equation 1, while equations 2 and 3 illustrate how LV correlations may be obtained by defining a scaling matrix D as the inverse square root of the diagonal elements of the latent variable covariance matrix ( $COV_{LL}$ ).

Equation 4, on the other hand, provides the formula for obtaining the MV/LV cross correlation matrix  $F_s$  (also called the factor structure matrix), while equation 5 provides the well known formula for obtaining the matrix of MV/LV regression coefficients ( $F_p$ ) from estimates of  $F_s$ .

Equations 6a and 6b illustrate alternative computational formulae for obtaining estimates of the MV correlation matrix ( $\hat{R}_{yy}$ ) depending on whether one has a matrix of MV/LV correlations ( $F_s$ ) or MV/LV regressions ( $F_p$ ). Equation 7, on the other hand, simply indicates that estimates of the residual correlation matrix ( $R_{EE}$ ) may be obtained by subtracting  $\hat{R}_{yy}$  from the original correlation matrix.

To summarize then, the full computational sequence for obtaining  $\hat{R}_{yy}$  would typically consist of the following equations:

- (1)  $A'R_{yy}A = COV_{LL}$
- (2)  $D = \text{DIAG}(COV_{LL})^{-\frac{1}{2}}$
- (3)  $DA'R_{yy}AD = R_{LL}$
- (4)  $R_{yy}AD = F_s$
- (6b)  $\hat{R}_{yy} = F_s R_{LL}^{-1} F_s'$
- (7)  $R_{EE} = R_{yy} - \hat{R}_{yy}$

### Estimated Correlations From a Full MV/LV Structural Model

Once an appropriate measurement model has been obtained by the PLS algorithm, two-stage least squares (2SLS) parameter estimates of the hypothesized structural relationships among latent variables are routinely generated by utilizing the covariance information contained in the LV correlation matrix ( $R_{LL}$ ). The following equations may then be utilized to produce estimates of the MV correlations from a given set of LV structural parameter estimates:

$$(8) L = [\eta; \xi]$$

$$(9) R_{LL} = \begin{bmatrix} C_{\eta\eta} & C_{\eta\xi} \\ C_{\eta\xi}' & \phi \end{bmatrix} \quad \text{where } \phi = \text{COV}(\xi, \xi) \text{ for standardized } \xi\text{'s}$$

$$(10) \eta = \eta B_o + \xi \Gamma + \zeta = [\eta; \xi] \begin{bmatrix} B_o \\ \Gamma \end{bmatrix} + \zeta$$

$$(11) \begin{bmatrix} \hat{B}_o \\ \hat{\Gamma} \end{bmatrix} = \text{2SLS (LV Structural model)}$$

$$(12) \hat{\eta} = \hat{\xi} \hat{\Gamma} (I - \hat{B}_o)^{-1} + \zeta (I - \hat{B}_o)^{-1} = \xi P + \zeta (I - \hat{B}_o)^{-1}$$

where  $P = \hat{\Gamma} (I - \hat{B}_o)^{-1}$

$$(13) \hat{\Psi} = \text{COV}(\hat{\zeta}, \hat{\zeta}) = \hat{B}' C_{\eta\eta} \hat{B} - \hat{B}' C_{\eta\xi} \hat{\Gamma} - \hat{\Gamma}' C_{\eta\xi}' \hat{B} + \hat{\Gamma}' \phi \hat{\Gamma}$$

$$(14) \hat{R}_{LL} = \begin{bmatrix} \hat{C}_{\eta\eta} & \hat{C}_{\eta\xi} \\ \hat{C}_{\eta\xi}' & \phi \end{bmatrix} = \begin{bmatrix} P' \phi P + B^{-1} \hat{\Psi} B^{-1} & P' \phi \\ \phi P & \phi \end{bmatrix}$$

where  $B = (I - \hat{B}_o)$

$$(15) \hat{R}_{yy} = F_s' R_{LL}^{-1} F_s$$

Definition of Symbols

- $L$   
 $n \times k$  - represents the actual matrix of latent variables which has been partitioned into two subsets: (1) The set of endogenous variables represented by  $\eta$  which is  $N \times p$ , and (2) the set of exogenous predetermined variables represented by the  $N \times q$  matrix  $\xi$ , with  $N$  being equal to the number of observations and with  $k = p + q$ , the number of endogenous and exogenous variables respectively.
- $B_0$   
 $p \times p$  - is the matrix of structural parameters representing hypothesized causal relationships among the endogenous latent variables of  $\eta$ .
- $\Gamma$   
 $q \times q$  - is the matrix of structural parameters representing hypothesized causal influences of the exogenous latent variables (the  $\xi$ 's) on the  $\eta$ 's.
- $\hat{B}_0, \hat{\Gamma}$  - represent structural estimates of  $B_0$  and  $\Gamma$  respectively.
- $\xi$   
 $N \times p$  - is the matrix of  $p$  disturbance terms which represent the "errors in equations" for estimating the  $\eta$ 's from the given latent variable structural equations.
- $\Psi$   
 $p \times p$  - is the matrix of covariance among the  $\xi$ 's.

$\hat{\eta}, \hat{\Psi}, \hat{R}_{LL}$  - are estimates of the endogenous variables, the  $\xi$  covariance matrix, and the standardized LV covariance (i.e. correlation) matrix respectively.

The Generating Equations

Equation 8 indicates a partitioning of the latent variables into two sets depending on whether they are (1) the ultimate causal sources in the model under consideration (i.e., the "exogenous"  $\xi$ 's) or (2) the mediating and/or dependent variables in the model (i.e., the "endogenous"  $\eta$ 's). Equation 9 indicates a similar partitioning of the latent variable covariance matrix  $R_{LL}$ . It is important to note, however, that this LV covariance matrix is the same covariance matrix obtained during the previous measurement model estimation phase described in the last section.

Equation 10 illustrates two equivalent forms of the general latent variable structural equation model. Equation 11, on the other hand, simply indicates that, given this particular model, we used the two-stage least squares estimates of the structural parameter matrices  $B_0$  and  $\Gamma$ . The PLS algorithm also provides Fixed Point parameter estimates. However, for our particular model these estimates were identical to the 2SLS estimates, and thus, the 2SLS estimates were used during further analysis.

For completeness, equation 12 has been included in order to describe the reduced form "equation for estimating the endogenous LV's from both the exogenous LV's ( $\xi$ 's) and the structural equation disturbance terms ( $\zeta$ 's). Equation 14, however, is the equation that we actually used for generating an estimate ( $\hat{R}_{LL}$ ) of the LV covariance matrix  $R_{LL}$  from estimates of the LV structural parameters. Equation

13, on the other hand, provides a formula for obtaining estimates of the disturbance covariance matrix ( $\hat{\Psi}$ ), which is necessary in order to perform the calculations indicated in equation 14. This formula for  $\hat{\Psi}$  was suggested by Fox (1979).

Equation 15 is the latent variable structural equation version of equation 6b. The only difference between the two is that  $R_{LL}$  is replaced by  $\hat{R}_{LL}$  which is dependent on the estimated structural parameter matrices  $\hat{B}_0$  and  $\hat{\Gamma}$ . Once  $\hat{R}_{yy}$  has been generated, the residual correlation matrix,  $R_{EE}$ , may be obtained by subtracting  $\hat{R}_{yy}$  from  $R_{yy}$  as in equation 7.

In summary, then, the full computational sequence for obtaining LV structural estimates of  $R_{yy}$  would typically consist of the following equations:

$$(9) R_{LL} = \begin{bmatrix} C_{\eta\eta} & C_{\eta\xi} \\ C_{\eta\xi}' & \phi \end{bmatrix}$$

$$(11) \begin{bmatrix} \hat{B}_0 \\ \hat{\Gamma} \end{bmatrix} = \text{2SLS (LV Structural model)}$$

$$(13) \hat{\Psi} = \hat{B}'C_{\eta\eta}\hat{B} - \hat{B}'C_{\eta\xi}\hat{\Gamma} - \hat{\Gamma}'C_{\eta\xi}'\hat{B} + \hat{\Gamma}'\phi\hat{\Gamma}$$

$$(14) \hat{R}_{LL} = \begin{bmatrix} C_{\eta\eta} & C_{\eta\xi} \\ C_{\eta\xi}' & \phi \end{bmatrix} = \begin{bmatrix} P'\phi P + \hat{B}^{-1'}\hat{\Psi}\hat{B}^{-1} & P'\phi \\ \phi P & \phi \end{bmatrix}$$

$$(15) \hat{R}_{yy} = F_s \hat{R}_{LL}^{-1} F_s'$$

$$(7) R_{EE} = R_{yy} - \hat{R}_{yy}$$

## Reference

- Fox, J. Solving simultaneous equations in two-stage least-squares. In K. Schuessler (Ed.) Sociological Methodology 1979. San Francisco: Jossey-Bass, 1979.



**END**