AN ECONOMETRIC INVESTIGATION OF PRODUCTION COST FUNCTIONS FOR LAW ENFORCEMENT AGENCIES

by

J.M. Heineke

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CENTER FOR THE ECONOMETRIC STUDIES OF CRIME AND THE CRIMINAL JUSTICE SYSTEM

Hoover Institution

Stanford University

Abstract

In this paper we adopt the economic model of an optimizing firm as a framework for characterizing the production structure of a sample of medium sized U.S. law enforcement agencies. Unlike previous studies we begin with a second order approximation to an arbitrary multi-output-multi-input production possibilities function which permits us to test a number of hypotheses which have been implicitly maintained in earlier work. Of particular interest are our findings that the decisions of police administrators are consistent with cost minimization and that outputs are very definitely joint--thereby effectively precluding estimation of separate production and/or cost functions for the different outputs of police agencies. In addition, we strongly reject the hypothesis of constant returns to scale and find that scale economies vary considerably with activity levels. Our sample also supported the hypothesis that a consistent index of burglary, robbery and larceny solutions can be calculated which would permit using the aggregate, say, "non-automobile thefts," in decision making contexts without loss of information.

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In this paper we study the relationship between costs, input prices and activity levels in a sample of approximately thirty medium sized city police departments for the years 1968, 69, 71 and 73. Our interest lies in determining the functional structure of law enforcement production technology.

Since efficient allocation of resources to activities requires knowledge of relative incremental costs for the activities involved, we are particularly interested in determining marginal cost functions for, and rates of transformation between the various outputs. Since past studies have adopted functional specifications which have implicitly maintained strong hypotheses about the underlying technology, we adopt a quite general functional specification which permits testing the appropriateness of these hypotheses. In a more general context we model and estimate the structure of production for a multiple output-multiple input firm in a manner which places few restrictions on first and second order parameters of the underlying structure.

Introduction

One question which arises immediately in any discussion of cost or production functions associated with law enforcement agencies concerns the

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appropriate measure of "output." Clearly police departments produce multiple outputs (services) for a community, ranging from directing traffic, quieting family squabbles, and providing emergency first aid, to preventing crimes and solving existing crimes. In this study we view police output as being of essentially two types: (1) general service activities as epitomized by the traffic control and emergency first aid care functions of police departments; and (2) activities directed to solving existing crimes. Strictly speaking, "solving existing crimes" is an intermediate output with deterrence or prevention of criminal activity being the final product. But due to the difficulty of measuring crime prevention we use the number of "solutions" by type of crime as output measures.[1]

In the past few years a number of authors have, to one degree or another, addressed the problem of determining the structure of production in law enforcement agencies. Since under certain rather mild regularity conditions there exists a duality between cost and production functions, either the cost function or the production function may be used to characterize the technological structure of a firm. The studies of Chapman, Hirsch and Sonenblum (1975), Ehrlich (1970, 1973), Votey and Phillips (1972, 1975) and Wilson and Boland (1977) all proceed by estimating production functions while Popp and Sebold (1972) and Walzer (1972) estimate cost functions. It is of some interest to briefly review the findings of these authors.

Chapman, Hirsch and Sonenblum estimate a rather traditional production function, at least from a theoretical point of view. All police outputs are collapsed into one aggregate, which is then regressed on input use levels utilizing data from the city of Los Angeles for the years 1956-70.

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[1]See Chapman, Hirsch and Sonenblum (1975) for an attempt to measure crime prevention as an output of police agencies.

They find strongly increasing returns to scale--often a two to four percent output response to a one percent change in input usage.

Ehrlich (1970, 1973) also uses an aggregate solution rate as the output measure, but instead of employing traditional input measures he regresses the aggregate solution rate on per capita expenditures on police, the aggregate offense rate and a series of exogenous ("environmental") variables. The expenditure variable is, of course, an index of overall input use levels while the aggregate offense rate is included to measure the effects of "crowding" or capacity constraints on output. This is a substantial departure from a neoclassical approach in which the shape of the production function itself will reflect diminishing returns as capacity is pressed. But it is a specification that has been widely adopted by those who have followed Ehrlich. (For example, see Vandaele (1975) or Votey and Phillips (1972, 1975).) Using per capita expenditures to measure the scale of output, Ehrlich finds that a one percent increase in expenditures per capita leads to much less than a one percent increase in the solution rate.

We should point out that two different arguments have been used for including the offense level in police agency production functions. In addition to the argument based upon police resource capacities, some authors have justified inclusion of the offense level in the production function using what is essentially a "fisheries argument." Viz., that the total number of fish in the ocean is a determinant of the number caught. So if the number of offenses is high, then <u>ceteris paribus</u>, it should be easier to obtain a solution than if there are but few offenses. Obviously, the argument goes, if there are no offenses there can be no solutions. But this is really not the question. The question is whether in the neighborhood of observed solution levels, changes in the total number of offenses would change solution levels.

Whichever rationale is used, the neoclassical production function is modified and written as $y = f(v_1, v_2, \ldots, v_m, O)$, where y is the number of solutions, v_i is the level of utilization of input i and O is the number of offenses. One means of testing the appropriateness of this specification is to assume that O does not belong in the production function and then estimate the function $y/O = f(v_1, v_2, \ldots, v_m)O^{\gamma}$, where y/O is the solution <u>rate</u>. If γ is significantly different from minus unity, the offense level probably influences solution levels. If not, one has some evidence that the production function for solutions is independent of the level of offenses.

Votey and Phillips (1975) report three estimates of the production function $y/O = \alpha v^{\beta} O^{\gamma}$. Using their reported parameter estimates and standard errors,one cannot reject the hypothesis that $\gamma = -1$ in any one of the estimated equations at the .05 level. In addition, Ehrlich's (1973) estimate of γ is -.908 which again is not significantly different from minus unity. We conclude, at least tentatively, that the production of solutions does not depend upon offenses and do not consider the matter further in this study.

Votey and Phillips (1972) estimate production functions which link solution rates for the property crimes of auto theft, burglary, larceny and robbery to input usage. As with Ehrlich and Vandaele, the authors include the level of offenses as an argument in the production function along with more traditional input measures.[2]

The Wilson and Boland study is similar to the work of Votey and Phillips (1972), in that they study the production of solutions to several

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[2]This is primarily an expository paper and only graphical analyses of the estimated functions are reported. Hence it is not possible to perform tests of the sort discussed in the previous paragraphs.

property crimes. But instead of input levels as determinants of solutions, they utilize a "capacity" variable and variables meant to account for productivity differences between departments. Here as with Vandaele and Votey and Phillips (1972), the authors cannot address the question of scale economies due to the fact that only a subset of all outputs are included in these studies.

Finally, both Popp and Sebold, and Walzer estimate cost functions and attempt to measure scale economies. The former use population size in the police jurisdiction as their measure of "scale" along with a large number of demographic and environmental variables to estimate the per capita costs of police service. Given the appropriateness of these variables for explaining costs, the authors find diseconomies of scale throughout the entire range of population sizes. Of course the population variable provides a considerably different concept of scale than economists are accustomed to considering, and in fact, Walzer has argued that population size is a poor measure of scale for several reasons--the most important being a tendency on the part of police administrators to determine manpower needs as a proportion of population size. In such a case there is obviously a strong bias toward constant returns to scale. In his study Walzer recognizes that offenses cleared, accidents investigated, etc., all make up the output of a police department. But instead of estimating a multiple output cost function, he creates an "index of police service" by collapsing all outputs into one.[3] The estimated cost function contains the offense rate as an argument in addition to measures of input prices, <u>input usage</u> and several variables meant to pick up externally determined differences in productivity. Using the service index to measure

[3]The weights used are average times spent on each type of activity.

output Walzer finds evidence of economies of scale, although they seem to be rather slight. Interestingly enough he also finds that input costs are not significantly related to overall production costs.

Outline of the Paper

A number of strong hypotheses concerning the production structure of law enforcement agencies have been implicitly maintained in the studies we have sketched. First, the arguments entering cost and production functions have for the most part differed considerably from what one would expect from classical production theory. In addition, in the one case where input costs do enter the cost function (Walzer), linear homogeneity in input costs has not been imposed on the estimated cost function. One possible explanation for these deviations from classical production and cost specifications is that classical theory, and cost minimizing behavior in particular, is not capable of explaining observed choices in public agencies. While this is a plausible hypothesis, it should be tested rather than maintained.[4]

Second, each of the estimated production functions upon which we have reported is either linear or linear logarithmic. Such functions may be viewed as first order approximations to an arbitrary production function. It is well known that first order approximations severely restrict admissable patterns of substitution among inputs and admissable rates of transformation among outputs as well as having other undesirable empirical

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[4]This hypothesis is explicit in Wilson and Boland, p. 8, who state, "In our view, police departments do not behave in accordance with the economic model of the firm."

implications.[5] An additional problem with linear logarithmic production or cost functions arises if one is interested in determining the extent of scale economies, since these functions do not permit scale economies to vary with output. On a related point, we noted above that each of the production studies surveyed included the offense rate or level as an argument. A possible explanation for this inclusion might be based upon the restrictiveness of the chosen functional forms and a consequent attempt on the part of the authors to provide output responses which do vary with the scale of operation, in functions which do not naturally possess this property. For these reasons and others we adopt a second order approximation to the underlying cost and production structure thereby leaving the various elasticity measures of common interest free to be determined by the data.[6]

Third, the Chapman, Hirsch and Sonenblum, Walzer and Ehrlich studies all utilize a single output aggregate. If the results of such aggregate studies are to be used for decision purposes, it is desirable that the aggregate measure be a consistent index over all police outputs. In what follows we estimate a multiple output cost function and test whether the various subsets of outputs may be consistently aggregated into single categories.

Fourth, the Wilson and Boland, Votey and Phillips (1972, 1975) and Vandaele studies each implicitly maintain the hypothesis of nonjoint outputs by estimating separate production functions for different types of solutions.

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[5]For example, linear logarithmic production functions imply input expenditure shares which are independent of the level of expenditure, while linear production functions imply perfect input substitutability and consequently rule out internal solutions to the cost minimization problem.

[6]In the Popp and Sebold, and Walzer studies the production cost function is specified to be quadratic in the scale argument although all other second order parameters are restricted to be zero.

Again, instead of maintaining this hypothesis we estimate a multiple output function and then test the nonjointness hypothesis.

To summarize, in this study we characterize the structure of production in a combined cross section and time series analysis of U.S. police departments, test for the existence of consistent aggregate indices of police output, for nonjointness of output, and for consistency of our estimated equations with the optimizing behavior of classical theory. In addition, we calculate (1) marginal and average cost functions for solutions to the property crimes of burglary, robbery, larceny and motor vehicle theft, and for solutions to crimes against the person; (2) marginal rates of transformation between these activities; and (3) an estimate of scale economies based upon the response of total cost to simultaneous variation in all police outputs.

Theoretical Background

The following definitions and concepts are used in what follows.

Let $F(y,v) = 0$ represent a "well behaved" production possibility frontier, where y is an n vector of outputs and v is an m vector of inputs. In addition, let $C(y,w)$ be the associated production cost function, where w is an m vector of input prices. It is well known that the cost function is a positive, increasing function of y and w. Furthermore, it can be shown that proportional increases in input prices, w, cause equi-proportional increases in cost, that is $C(y,w)$ is linear homogeneous in w. This property of $C(y,w)$ is due to the cost-minimizing behavioral hypothesis of neoclassical models of the firm.

Other properties of the cost function are determined by the technological conditions of production. Among the properties of interest are those of nonjointness of outputs, and the existence of so called "consistent" aggregates of outputs. More precisely, a technology is <u>nonjoint</u>, if the production function $F(y,v)$ can be decomposed into n separate production functions $y_i = f_i(v)$, with the property that $f_i(v)$ is independent of y_j, $i \neq j$. Therefore, in order to show that a technology in nonjoint, the functions $f_i(\cdot)$ must exist and be free of any economies or diseconomies of jointness. As Hall (1973) has pointed out, this does not require physically separate processes producing the various outputs, nor does the fact that two or more outputs are produced in the same plant rule out nonjointness. It is obvious that the existence of nonjoint outputs dramatically simplifies the estimation procedure. In terms of production cost, nonjoint outputs imply the cost function can be written as the sum of individual costs, i.e. $C(y,w) = C_1(y_1,w) + C_2(y_2,w) + \ldots + C_n(y_n,w)$.

The problem of aggregating outputs is concerned with finding <u>consistent</u> means of collapsing several outputs into one output index. Aggregation is said to be consistent if the solution to a problem at hand is identical regardless of whether one uses aggregate indices or the micro level variables.

There are at least two methods of aggregation. Hicks introduced the idea that if the prices of a group of goods always change in the same proportion, that group of goods can be treated as a single commodity. Another possibility is based upon separability of a group of outputs from other groups. In particular, outputs y_i and y_j are separable from other outputs and inputs if and only if the marginal rate of transformation between outputs i and j are independent of all other outputs and inputs.

In other words, the firm's decision as to the optimal mix of outputs i and j is not affected by the level of other outputs or by input usage. If a group of outputs is separable, and in addition, possesses a particular functional structure, so called homothetic separability, a consistent aggregate index for the group exists.[7]

Motivation of Agencies

In this section we provide a framework within which the structure of law enforcement production technology could be estimated. The model is essentially a value maximization model and implies that input decisions are reached in cost minimizing manner. We assume that police administrators, either implicitly or explicitly, assign "seriousness" weights to crimes by type and use these weights along with the costs of solving crimes by type to determine the solution mix. This might be termed a "bounty hunter" model of police decision making since resources are allocated to solutions by type as if police remuneration were proportional to the "value" of solved crimes and assumes that police decision makers are primarily interested in solutions and not deterrence.[8] We believe that on a day to day basis a strong argument can be made that police administrators are primarily concerned with solutions and not deterrence and that for property crimes average values stolen are likely to be reasonable approximations to the weights used in allocating resources to solving property crimes.

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[7] For more detail see Darrough and Heineke (1977).

[8] Michael Block has suggested this terminology which is particularly descriptive of the model.

Using P_i to represent the value to police of a solution of a crime of type i, the police agency's decision problem is

$$\text{(1)} \qquad \max \sum_{i=1}^{n} P_i y_i - C(y,w).^{8'}$$

Decision problem (1) provides the familiar system

$$\text{(2)} \qquad P_i - \partial C/\partial y_i = 0, \qquad i = 1, 2, \ldots, n$$

which may be used as a basis for estimating C(y,w). Equation (2) instructs police administrators to allocate resources to solving crimes of type i until the marginal cost of a type i solution is equal to the assigned weight, P_i. Note that if C(y,w) is approximated with a polynomial in y and w, equations (2) alone will not be sufficient to determine the cost function. This can be remedied by including C(y,w) itself in the system to be estimated. In which case

$$\text{(3)} \qquad \begin{aligned} P_i - \partial C/\partial y_i &= 0, \qquad i = 1, 2, \ldots, n \\ C - C(y,w) &= 0 \end{aligned}$$

becomes the system of interest. In the circumstances we have outlined it is reasonable to assume that values P_i are determined jointly by the activities of police and offenders in earlier periods--i.e., P_i are predetermined. Assuming that input costs are exogenous, equations (3) determine the n endogenous solution levels as functions of exogenous and predetermined variables.

One problem in implementing this system in an econometric context is obvious: The weights to be given the various types of solution are at

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[8'] Of course there is a constraint on the decision problem which we have not taken into account: Viz., that $C(y,w) \leq A$, where A is the agency's budget for the period.

best difficult to obtain. But as we have indicated above, in the case of property crimes average values stolen probably provide reasonable approximations to the seriousness of these crimes in the eyes of the police. Although for the case of "crimes against the person," e.g., homocide, rape and assualt, no such convenient measure is available.

One method of dealing with this problem is to assume that property crime solutions are separable from all other police activities. As we indicated above, this is equivalent to assuming that marginal rates of transformation (MRT) between solutions to all pairs of property crimes be invariant to the level of nonproperty crime solutions and to the level of other police services provided, e.g., traffic control, emergency first aid, etc. In this case, it can be shown that there exists functions C* and f such that the cost function may be written as

$$(4) \qquad C = C^*(f(y_1, \ldots, y_p, w), y_{p+1}, \ldots, y_n, w)$$

where $y_1, \ldots, y_p$ represent solutions to crimes against property and $y_{p+1}, \ldots, y_n$ represent solutions to crimes against the person and the service activities performed by police.

Equations (3) now become

$$(5) \qquad P_i - \partial C^*/\partial y_i = 0, \qquad i = 1, 2, \ldots, p$$

$$C - C^*(f(y_1, y_2, \ldots, y_p, w), y_{p+1}, \ldots, y_n, w) = 0$$

and are estimated below for the case of four property crimes, burglary, robbery, motor vehicle theft and larceny, an aggregate of crimes against the person and an aggregate police service indicator.[9]

The Translog Model

From an econometric point of view equation system (5) is only of limited interest until a specific functional form has been assigned to the cost function C*(y,w). The primary concern in choosing a functional form for C* is that the chosen class of functions be capable of approximating the unknown cost function to the desired degree of accuracy. In widespread use in the literature in the past few years are the class of so called "flexible" functional forms which includes the generalized Leontief function, the generalized Cobb-Douglas function, the transcendental logarithmic function and many hybrids.[10] These functions may all be viewed as second order approximations to arbitrary production or cost functions and in particular place no restrictions on elasticities of substitution between inputs or elasticities of transformation between outputs and allow returns to scale to vary with the level of output. We have chosen to approximate C*(y,w) with the translog function due primarily to the fact that most past studies of law enforcement agency production technology have adopted linear logarithmic functions which are special

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[9] An alternative approach to estimating the production structure of law enforcement agencies would be to assume that police take as given a vector of outputs which is minimally acceptable to the community and provide at least that level of service at minimum cost. Among other reasons for choosing the value maximization framework over the cost minimization framework is that the former explicitly addresses the output mix problem rather assuming that this decision is exogeneous. See Darrough and Heineke [1977] for more detail.

[10] See Diewert (1971, 1973, 1974) and Christensen, Jorgensen and Lau (1971, 1973, 1975).

cases of the translog function.

The translog cost function may be written as

(6) $$\ln C(y,w) = a_0 + \sum_1^n a_i \ln y_i + \sum_1^m b_i \ln w_i + \frac{1}{2}\sum_1^n \sum_1^n \alpha_{ij} \ln y_i \ln y_j$$

$$+ \frac{1}{2}\sum_1^m \sum_1^m \beta_{ij} \ln w_i \ln w_j + \sum_1^m \sum_1^n \gamma_{ij} \ln y_i \ln w_j .$$

It can be shown that second order parameters of this function must be symmetric if supply functions are to be well behaved, i.e., $\alpha_{ij} = \alpha_{ji}$ and $\beta_{ij} = \beta_{ji}$ and $\gamma_{ij} = \gamma_{ji}$ for all i and j. Our maintained hypothesis of separability (see equation 4) between property crime solutions and all other activities of the police agency implies the following restrictions on equation (6):

(7) $$\alpha_{ij} = 0, \qquad i = 1, 2, \ldots, p, \ j = p+1, p+2, \ldots, n.$$[11]

In general, hypotheses concerning the nature of production technology impose certain restrictions on the values of the parameters of the empirical cost function. In particular, the hypothesis of linear homogeneity of C(y,w) in inputs prices, which is an implication of cost minimizing behavior, imposes the following restrictions on the translog cost function:[12]

(8) $$\sum_i^m b_i = 1, \quad \sum_j^m \beta_{ij} = \sum_i^m \beta_{ij} = \sum_j^m \gamma_{ij} = 0.$$

If these restrictions are imposed, then proportional increases in input prices lead to equi-proportional increases in production costs. The hypothesis of cons-

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[11] An alternative means of imposing separability on the translog cost function exists, but is not pursued here. See Darrough and Heineke (1977) for more detail.

[12] Perhaps it is worth reemphasizing at this point that linear homogeneity of production costs in input prices is only necessary for cost minimizing behavior. For example, if inputs are always utilized in precisely fixed proportions, then production costs will be linearly homogeneous in w independent of the behavioral motivations of the firm. Of course if one accepts linear homogeneity but suspects a fixed proportion production structure, the latter is a testable hypothesis.

tant returns to scale implies:

$$(9) \qquad \sum_i^n a_i = 1, \quad \sum_j^n \alpha_{ij} = \sum_i^n \alpha_{ij} = \sum_i^n \gamma_{ij} = 0$$

and of course means that a given percentage change in all outputs leads to the same percentage change in production cost.

Another hypothesis of considerable interest is that of nonjointness of outputs. As we indicated above, if outputs are nonjoint one may estimate a separate cost function for each output. In terms of the translog cost function nonjointness of outputs means that all cross second order terms in y are zero, i.e.,

$$(10) \qquad \alpha_{ij} = 0, \qquad\qquad i, j = 1, 2, \ldots, n, \ i \neq j.$$

These restrictions and others on the production technology of law enforcement agencies are tested below.[13]

The Econometric Model

In this section we specialize the n output, m input production model to the model which is estimated and provide the stochastic specification needed for estimation. We had available for this study information on annual police budgets for the years 1968, 1969, 1971 and 1973 for a sample of approximately thirty medium size cities;[14] the average wages of officers by rank, the number of crimes of type i cleared by arrest ("clearances") and the average value stolen for each of the property crimes in the FBI index. The police budget and wage information was

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[13] See Darrough and Heineke [1977] and the accompanying references for further discussion of these restrictions.

[14] The largest city in our sample is Houston, Texas, (1,230,000), the smallest is Birmingham, Alabama (300,000). Mean population over the sample is 561,000.

gathered by the Kansas City Police Department and circulated for use by participating cities under the title of the Annual General Administrative Survey. The data on clearances and average values stolen are from unpublished sources at the FBI. We have used clearances by arrest for the seven FBI "index crimes" as our measures of "solutions." In particular, we have called burglary clearances (solutions), y_1, robbery clearances, y_2, motor vehicle theft clearances, y_3, and larceny clearances, y_4. We have used the aggregate number of homocide, rape and assault clearances to represent solutions to crimes against the person and have labeled this output, y_5. Finally, a very large component of the output of all law enforcement agencies are the rather mundane but important service functions--directing traffic, investigating accidents, breaking up fights, providing emergency first aid, etc. We group all such service functions together as y_6. The question is what to use to measure these activities. We have adopted the hypothesis that the quantity of services of the type we have been discussing is proportional to the size of the city in which the agency is located. This gives a cost function with six outputs and a still unspecified number of input prices.

We had available wage information on eight grades of police officers from patrolman to chief. As one might expect. these wage series are highly collinear. To test for the existence of a Hicksian price index, we computed correlation coefficients between the wages of the various ranks and found very high coefficients. For example, the correlation between wages of patrolmen and a weighted average of the wages of all other ranks is .955. Unfortunately, there does not appear to be a way of testing whether a sample correlation is significantly different from one since the distribution of this statistic is degenerate at that point. But with correlations this high it appears safe to assume the conditions for Hicks' aggregation are fulfilled and hence we use a weighted average of all police wages as an aggregate measure of unit labor costs, denoted w.[15]

The translog cost function of (6) above may now be written as

$$(11) \qquad \ln C^*(y,w) = a_0 + \sum_1^6 a_i \ln y_i + b\ln w + \frac{1}{2}\sum_1^6 \sum_1^6 \alpha_{ij} \ln y_i \ln y_j + \beta \ln w^2 + \sum_1^6 \gamma_i \ln w \ln y_i$$

where $\alpha_{15} = \alpha_{16} = \alpha_{25} = \alpha_{26} = \alpha_{35} = \alpha_{36} = \alpha_{45} = \alpha_{46} = 0$ due to the imposed separability of property crime solutions and from all other police activities. The restrictions on the cost function implied by linear homogeneity in input prices, constant returns to scale and nonjointness of outputs have been discussed above.[16]

Given the hypothesis of separability between property crime solutions and all other police activities there are a total of eleven possible groupings of property crime solutions which might be considered for

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[15]Cost and wage series have been deflated using an index based upon BLS Intermediate Family Budget data. (See B.L.S. Bulletins No. 1570-7 and the Monthly Labor Review.

[16]Linear homogeneity of C* in w would impose the following restrictions: $b = 1$, $\beta = 0$, $\sum_1^6 \gamma_i = 0$, while constant returns to scale imply $\sum_1^6 a_i = 1$, $\sum_i^6 \alpha_{ij} = \sum_j^6 \alpha_{ij} = 0$, and $\sum_1^6 \gamma_i = 0$. If property crime solutions are nonjoint then $\alpha_{ij} = 0$, $i,j = 1, 2, 3, 4$, $i \neq j$. The latter imposes only six additional restrictions, due to symmetry of the α_{ij}.

indexing.[16] Our question here is not whether an index exists in any of these cases, because an index can always be found, but whether a <u>consistent</u> index exists.[17] It is important to keep in mind that the existence of a separable group of outputs does not in general imply existence of a consistent index for the group.

For the translog cost function, it is convenient to express equations (5) in the following "value share" form:

$$(12)\qquad P_i y_i / C^* = a_i + \sum_1^6 \alpha_{ij} \ln y_j + \gamma_i \ln w, \qquad i = 1, 2, 3, 4$$

$$\ln C^* = a_0 + \sum_1^6 a_i \ln y_i + b \ln w + \tfrac{1}{2}\sum_1^6 \sum_1^6 \alpha_{ij} \ln y_i \ln y_j + (\beta/2) \ln w^2 + \sum_1^6 \gamma_i \ln w \ln y_i$$

where $\alpha_{ij} = 0$, $i = 1, 2, 3, 4$, $j = 5, 6$ and $\alpha_{ij} = \alpha_{ji}$, for all i and j. (The first four equations here give the value of y_i solutions to property crime i as a proportion of total police expenditures.) The next step in implementing the econometric version of the model is to provide a stochastic frameowrk for equations (12). We do this by appending classical additive disturbances to each of the five equations in the model. These disturbances arise either as a result of random error in the maximizing behavior of police administrators, or as a result of the fact that the

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[16] These groups are: (y_1,y_2), (y_1,y_3), (y_1,y_4), (y_2,y_3), (y_2,y_4), (y_3,y_4), (y_1,y_2,y_3), (y_1,y_2,y_4), (y_1,y_3,y_4), (y_2,y_3,y_4) and (y_1,y_2,y_3,y_4).

[17] An example of such a question is whether it is possible to aggregate burglary, robbery and larceny solutions into a composite category such as "non automobile theft" solutions so that the aggregate index may be used for decision purposes without loss of information from the micro level variables.

translog function provides only an approximation of the "true" underlying production structure. We assume that noncontemporaneous disturbances are uncorrelated both within and across equations. We make no other assumptions about the distribution of disturbances other than they be uncorrelated with right hand variables in each equation.[18]

Empirical Results

We have fitted the five equations of system (16) under the stochastic specification outlined above. There were 111 observations available for estimating each equation in the system. Since no assumption has been made concerning the distribution of disturbances, our estimation procedure may be thought of as multiequation, nonlinear least squares. In the computations we used the Gauss-Newton method to locate minima. The results of estimation are presented in Table I.

The estimates reported in column two contain no restrictions other than symmetry, and entail estimating twenty-eight parameters. Given the primarily cross section nature of the data, the model fits quite well with R^2 figures of .74 for the cost function and .36, .13, .46, and .29 for the value of solution equations $P_i y_i / C^*$, $i = 1, 2, 3, 4$, respectively.

In column three, we report estimates of the model with homogeneity in input prices imposed. As we have noted previously, cost minimizing input

[18] The latter is in fact a rather strong assumption, but one which is automatically satisfied under the assumptions we have adopted as long as errors over the years in our sample are serially independent.

TABLE I
Parameter Estimates for Five Cost Models

Parameter	Unrestricted Model	Homogeneity in Input Prices	Homogeneity and Nonjoint Outputs	Homogeneity and Linear Logarithmic Costs	Homogeneity and Constant Returns to Scale
a_o	-108.68 (27.23)	-98.899 (7.512)	-75.949 (2.190)	-4.469 (1.092)	-.7190 (1.332)
a_1	-.0049 (.0478)	-.0542 (.0168)	-.1326 (.0127)	.0292 (.0016)	.0053 (.0114)
a_2	.0244 (.0118)	.0203 (.0110)	.0129 (.0108)	.0065 (.0003)	.0314 (.0109)
a_3	.3262 (.0679)	.2989 (.0615)	.2378 (.0603)	.0459 (.0026)	.3956 (.0646)
a_4	.0252 (.0293)	.0031 (.0205)	-.0467 (.0203)	.0198 (.0009)	.0378 (.0190)
a_5	1.657 (1.682)	-2.118 (1.084)	-.4037 (.4853)	.2448 (.0376)	.4127 (.4088)
a_6	16.016 (.5917)	16.38 (.6349)	12.259 (.1848)	.9113 (.0902)	.1170 (.4147)
b	.7123 (7.393)	1	1	1	1
α_{11}	.0237 (.0020)	.1199 (.0591)	.0296 (.0561)		.0206 (.0014)
α_{22}	.0033 (.0005)	.0034 (.0005)	.0032 (.0003)		.0032 (.0005)
α_{33}	.0287 (.0022)	.0284 (.0022)	.0294 (.0019)		.0189 (.0019)
α_{44}	.0125 (.0009)	.0125 (.0009)	.0119 (.0009)		.0115 (.0007)
α_{55}	.0448 (.0528)	.0177 (.0524)	.0504 (.0441)		.0209 (.0621)
α_{66}	-1.451 (*)	-1.2711 (*)	-.0005 (*)		.0209 (.0621)
α_{12}	-.0022 (.0006)	-.0023 (.0006)			-.0277 (.0006)
α_{13}	-.0049 (.0016)	-.0051 (.0017)			-.0115 (.0015)
α_{14}	-.0053 (.0010)	-.0055 (.0010)			-.0063 (.0008)
α_{23}	-.0002 (.0005)	-.0002 (.0005)			-.0013 (.0005)
α_{24}	.0013 (.0004)	.0013 (.0004)			.0008 (.0004)
α_{34}	-.0022 (.0010)	-.0022 (.0010)			-.0061 (.0008)
α_{56}	.0996 (.0882)	.1097 (.0884)			-.0209 (.0621)

TABLE I (Continued)

Parameter Estimates for Five Cost Models[19]

Parameter	Unrestricted Model	Homogeneity in Input Prices	Homogeneity and Nonjoint Outputs	Homogeneity and Linear Logarithmic Costs	Homogeneity and Constant Returns to Scale
β	-.1471 (.5356)				
γ_1	-.0082 (.0071)	-.0963 (.0590)	-.0077		.0017 (.0015)
γ_2	-.0048 (.0017)	-.0041 (.0016)	-.0039 (.0016)		-.0034 (.0016)
γ_3	-.0617 (.0100)	-.0571 (.0090)	-.0574 (.0088)		-.0496 (.0097)
γ_4	-.0087 (.0043)	-.0050 (.0028)	-.0041 (.0029)		-.0041 (.0068)
γ_5	-.4615 (.1809)	.1029 (.0586)	.0286 (.0562)		-.0068 (.0231)
γ_6	.4798 (*)	.0598 (.0236)	.0445 (.0236)		.0622 (.0255)
ln of likelihood function	1654.57	1652.31	1625.77	1483.57	1613.23

[19]Standard errors are in parentheses.

*Collinearity problems prevented estimation of this standard error.

decisions imply a production cost function with this property and for this reason we may consider a test of the fit of the homogeneous model as a test of the consistency of the data with cost minimizing behavior.

Homogeneity in input prices reduces the number of parameters to be estimated from twenty-eight to twenty-five (see footnote 16). Traditional R^2 statistics are .72 for the cost equation and .35, .12, .46 and .28 respectively for the value share equations.

In columns four, five and six are reported parameter estimates for the cases of nonjoint outputs, linear logarithmic costs and constant returns to scale, each conditional on cost minimizing behavior. In column four are the estimates with linear homogeneity of input prices and non-jointness of output imposed. These restrictions reduce the number of parameters to eighteen (see footnote 16). The linear logarithmic cost function (column five) was estimated primarily to contrast the functional form of the cost function presented in this paper with that implied by the linear logarithmic production functions which have been estimated in the majority of earlier papers. The total number of parameters to be estimated is now reduced to seven. The final column contains our estimate of the model with constant returns to scale imposed.

As we have noted, assumptions concerning the police production technology imply restrictions on the parameters of the estimated cost function. Testing the consistency of the data with various sets of restrictions is one of the goals of this study and may be accomplished by estimating the model under each set of restrictions and then by comparing the "fit" of the different versions of the model. For discriminating among several versions of the model, we use the test statistic

(13) $\lambda = \max L^{R}/\max L^{\overline{R}}$

where $\max L^{R}$ is the maximum value of the likelihood function for the model with restrictions R and $\max L^{\overline{R}}$ is the maximum value of the likelihood function without restriction. Minus twice the logarithm of λ is asymptotically distributed as chi-squared with number of degrees of freedom equal to the number of restrictions imposed. Logarithms of the likelihood function are given in Table I for each of the model specifications to be evaluated. Throughout, we choose a critical region based upon .01 level of significance.

We now report the results of statistical tests performed on the estimated models. These tests are of one of two types: Tests concerned with the implications of the behavioral hypothesis of cost minimization and tests concerned with evaluating the characteristics of police agency production. A natural sequence of tests would therefore be to first test the consistency of our sample with the cost minimizing behavior of police agencies and then to proceed to tests of the structure of production conditional on the outcome of the first test. Since cost minimizing behavior requires that C* be linearly homogeneous in w, we test this hypothesis first. Comparing the homogeneous model with the unrestricted model we find that minus twice the logarithm of the likelihood ratio is 4.52. Since there are but three restrictions imposed, we easily accept the hypothesis of a cost function which is linearly homogeneous in input prices. That is, the data in our sample of police departments are consistent with cost minimizing behavior.

Conditional on the cost minimizing behavior of police decision makers we next test the validity of the hypothesis of nonjoint outputs--a hypothesis which has been maintained in all past studies in which multiple outputs have been dealt with. Minus twice the logarithm of the likelihood ratio is 53.08. Since nonjointness entails seven additional restrictions, the hypothesis is resoundingly rejected. We conclude that one may not go about estimating separate production functions or separate cost functions for each of the outputs of police agencies. The interaction between outputs must be accounted for if one is to adequately characterize the structure of cost and production in this "industry."

It is instructive to contrast the linear logarithmic cost and production structure implied by these data, with our more general model. Columns three and four of Table I contain parameter estimates for the cost models which maintain homogeneity in prices, and a linear logarithmic production structure in addition to linear homogeneity in prices.[20] The fact that twice the logarithm of the likelihood ratio for this test is 337.48 is an accurate indication of the magnitude of the loss in explanatory power resulting from adopting the Cobb-Douglas functional form for C*.

We next test the hypothesis that the underlying production function exhibits constant returns to scale. The logarithm of the likelihood function associated with this model (linear homogeneity in input prices <u>and</u> outputs) is reported in Table I. According to footnote 16, linear homogeneity in outputs imposes seven additional restrictions on the model.[21] The value of the test statistic is 78.16 and hence these data lend no

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[20]Of course, linear logarithmic cost and production functions maintain the nonjointness hypothesis.

[21]Symmetry of the α_{ij} reduces the restrictions in the second set of equations in footnote 16 from thirteen to seven. Recall that $\sum\gamma_i = 0$ is already imposed.

support whatever to the constant returns hypothesis.[22]

We noted in the introduction that a number of past production and cost studies used aggregate measures of police output. Whether or not such a procedure is desirable depends upon the existence of a "consistent index" for the various outputs. We also noted that a consistent aggregate index might be obtained either via homothetic separability or via Hicks' aggregation theorem. According to the former, a consistent index can be obtained if "group functions" exist and are homothetic.[23] Hicks' theorem states that a subset of y may be treated as a single output if the values assigned to these outputs are perfectly correlated.

The model was estimated with separability imposed for each potential aggregate. (See footnote 17.) Only in the case of the aggregate (y_1, y_2, y_4) was the hypothesis of separability accepted. In addition we found that our tests led to acceptance of the hypothesis that the cost function itself is homothetic. We conclude that the aggregate (y_1, y_2, y_4) is homothetically separable from other police outputs and input prices and hence a consistent index of burglary, robbery and larceny solutions exists. Such an index could be used in place of the "micro" variables y_1, y_2, and y_4 in decision making without loss of information.[24]

Finally, we have calculated the correlation matrix for P to check for the possibility of a Hicksian aggregate. The correlations are r_{12} = .065, r_{13} = .065, r_{14} = .901, r_{23} = .197, r_{24} = .014 and r_{34} = .026. (Of course, such calculations permit testing only pairwise groupings of outputs in the first step.) It is interesting to note that the only candidates for a Hicksian index are y_1 and y_4 which are included in the group of outputs (y_1, y_2, y_4) for which we have concluded a consistent index does exist. The question is whether .901 is significantly different from 1.0. It is not possible to test this proposition, since the distribution of sample correlation coefficient is degenerate at 1.0. However, .901 seems

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[22]We find below that the hypothesis of constant returns to scale is accepted <u>at sample means</u>. Of course, and as our test indicates, this does not imply constant returns to scale throughout the relevant output region.

[23]If a group of outputs are separable from other outputs (and inputs), then the group function exists. For example, if y_1 and y_2 are separable, then the cost function C* may be written as $C^*(y,w) = C^{**}(h(y_1,y_2), y_3, y_4, \ldots, w)$, where h is the group function.

[24]See Darrough and Heineke (1977) for more detail on the testing procedure.

distant enough from 1.0 to conclude that y_1 and y_4 may not be treated as a single output.

Marginal Costs, Rates of Transformation and Returns to Scale

The marginal cost function for activity i is given by $\partial C^*/\partial y_i = (\partial \ln C^*/\partial \ln y_i)(C^*/y_i)$, i = 1, 2, 3, 4, 5, and may be calculated using the formula

$$(14) \qquad \partial C^*/\partial y_i = (a_i + \sum_1^6 \alpha_{ij} \ln y_j + \gamma_i \ln w)(e^{\ln C^*}/y_i), \quad i = 1, 2, \ldots, 5.$$

As indicated, (14) will be valid for each of the crime solving outputs, $y_1, y_2, \ldots, y_5$ but not for y_6. Recall that the sixth output was an aggregate of the "non-crime solving" services provided by police. Since we have postulated only that the production of this output is proportional to population size, it will be possible to determine $\partial C^*/\partial y_6$ only up to this factor of proportionality.

The rate of transformation of output i for output j gives the number of solutions to crimes of type i which must be forgone for an additional solution to a crime of type j, given fixed levels of all other outputs. Formally, the rate of transformation between outputs i and j may be written as $-\partial y_i/\partial y_j = (\partial C^*/\partial y_j)/(\partial C^*/\partial y_i)$, i, j = 1, 2, ..., 5, i ≠ j, and may be calculated using the formula

$$(15) \qquad -\frac{\partial y_i}{\partial y_j} = \frac{(a_j + \sum_1^6 \alpha_{jk} \ln y_k + \gamma_j \ln w) y_i}{(a_i + \sum_1^6 \alpha_{ik} \ln y_k + \gamma_i \ln w) y_j}, \quad i, j = 1, 2, \ldots, 5, \ i \neq j$$

As with marginal cost functions, it will not be possible to obtain transformation rates between output six and other outputs.

Traditional measures of scale economies (or diseconomies) are predicated on the single output firm and must be modified for use here. We measure scale economies as the inverse of the percentage response of costs to a small equal percentage change in all outputs. That is, if

$$(16) \qquad \varepsilon \equiv dC^*/C^* = \sum_{1}^{6} (\partial \ln C^*/\partial \ln y_i)(dq/q),$$

where dq/q is the percentage change in outputs, then $1/\varepsilon$ is the usual measure of economies of scale.[24] ε measures the percentage response of costs to an equal percentage change in all solutions and in the service output.[25]

Defining average cost functions for the various outputs presents something of a problem in the case of multiple output production structures. We have calculated the average cost of solutions of type i by evaluating

$$(17) \left[C^*(\bar{y}_1,\ldots,\bar{y}_i,\ldots,y_6,\bar{w}) - C^*(\bar{y}_1,\ldots,\min y_i,\ldots,\bar{y}_6,\bar{w})\right]/(\bar{y}_i - \min y_i)$$

where an overbar indicates a sample means and min y_i is the minimum sample value of y_i.[26] This approach holds input prices and all outputs, except y_i, constant and yields the average value of the increment in costs over the region between the minimum value of y_i and the mean of y_i.

In Table II we have evaluated the cost responsiveness function, ε,

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[24] E.g., if $dq/q = 1$, and $\varepsilon < 1$ at y^*, then the production function exhibits increasing returns to scale at the output mix y^*, etc.

[25] The proportionality between population size and y_6 causes no problem in calculating returns to scale since the percentage change in y_6 is equal to the percentage change in population size.

[26] $C^*(y, w) \equiv e^{\ln C^*(y,w)}$

TABLE II

Marginal and Average Costs of Outputs, Rates of Transformation and Cost Responsiveness Functions at Sample Means*

MC_1	$ 307.40	AC_1	$ 265.15	MRT_{12}	.737	MRT_{24}	.607
MC_2	$ 226.59	AC_2	$ 262.35	MRT_{13}	3.815	MRT_{25}	11.898
MC_3	$1172.67	AC_3	$1271.26	MRT_{14}	.448	MRT_{34}	.117
MC_4	$ 137.73	AC_4	$ 127.53	MRT_{15}	8.543	MRT_{35}	2.239
MC_5	$2626.15	AC_5	$4615.80	MRT_{23}	5.175	MRT_{45}	19.067
						ε	.884

*Standard errors were calculated for ε, and marginal cost functions at this point. Each was highly significant at the .01 level.

marginal cost functions, average cost functions and marginal rates of transformation between outputs at sample means.

We find that estimated marginal costs are lowest for solving larcenies at $137, followed by those for robbery at $226, burglary at $307, motor vehicle solutions at $1172 and solutions to crimes against the person at $2626.[27] Rates of transformation between outputs at sample means range from .11 between motor vehicle theft solutions and larceny solutions to nineteen between larceny solutions and solutions to crimes against the person. Hence, the estimated cost function predicts that on average it will be necessary to forego between eight and nine larceny solutions to solve one additional motor vehicle theft (at the mean) and approximately nineteen larceny solutions to solve an additional crime against the person. Similar interpretations hold for the other transformation rates. Unit costs of clearing larcenies are $127, followed by robbery at $262, burglary at $265, motor vehicle solutions at $1271 and solutions to crimes against the person at $4615. Comparing marginal cost estimates with associated average costs, indicates that marginal costs of solving robberies, auto thefts and crimes against the person are below average costs and hence unit costs are falling (at the sample mean) for these activities. Marginal costs are greater than average costs for solving burglaries and larcenies, indicating rising unit costs (at the sample mean) for these activities.

We have estimated the value of ε to be .884, which turned out to be not significantly different from unity. But as Figure 1 indicates, scale economies vary greatly over the sample with decreasing, then constant,

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[27] Of course the model insures that "on average" marginal costs are equal to values stolen. Notice that this does not imply that marginal costs evaluated at the mean are equal "on average" to values transferred. More importantly, our interest in this study is primarily in the structure of law enforcement production technology and hence not in local properties of marginal and average cost functions.

then increasing returns to scale as output levels increase. Sample values of ε range from 1.62 to .53. To the extent that small cities have few solution levels, it appears that "large" cities have technological advantages in the provision of police services.

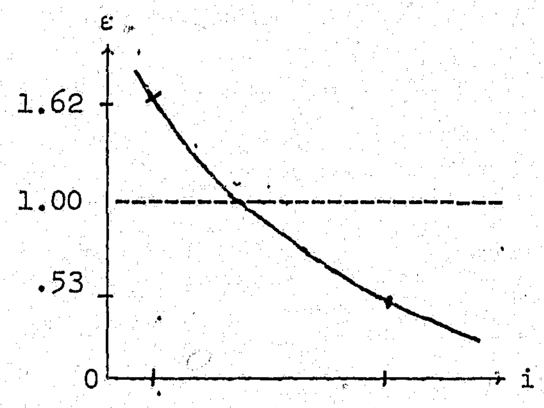

Figure 1

Cost Responses: All Police Activities

In interpreting this finding one should keep in mind that the cities in our sample range in size from approximately one third million to only a little over one million. Therefore one should not conclude that very large American cities experience increasing returns to scale in the provision of police services, since scale diseconomies may appear as city size continues to increase.[28]

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[28]In the past few years there has been considerable discussion concerning the share of the total police budget going to non-crime solving activities. All parties seem to agree that the share is high and has been increasing. For example, unpublished studies by the Vera Institute of Justice and the Cincinnati Institute of Justice indicate that police officers spend only about 15 to 20 percent of their time in crime solving activities. To provide additional information on this point, we have calculated $AC_6(\overline{y},\overline{w})\cdot\overline{y}_6/C(\overline{y},\overline{w})$ to measure the budget share of activity six--non-crime solving activities. (This calculation assumes that unit costs of these police services are approximately constant up to $\overline{y}_6$. See equation (17) above.) We find that the budget share of non-crime solving activities is slightly more than 80 percent at the sample mean--a result strikingly consistent with the studies mentioned.

Summary and Conclusions

In this paper we have adopted the economic model of an optimizing firm as a framework for characterizing the production structure of a sample of medium sized U.S. law enforcement agencies. Unlike previous studies we have begun with a second order approximation to an arbitrary multi-output-multi-input production possibilities function. This rather general functional specification has permitted us to test a number of hypotheses which have been implicitly maintained in earlier work. Of particular interest are the findings that, at least in our sample, the decisions of police administrators are consistent with cost minimization and that outputs are very definitely joint--thereby effectively precluding estimation of separate production and/or cost functions for the different outputs of police agencies. In addition, we strongly rejected the hypothesis of constant returns to scale and found that scale economies varied considerably with activity levels--which pointed up the inappropriateness of maintaining a Cobb-Douglass production structure in studies of law enforcement production technology. We then found that our sample supported the hypothesis that an index of burglary, robbery and larceny solutions can be calculated which would permit using

Finally, we calculated returns to scale, marginal costs, average costs and marginal rates of transformation at the sample mean. As always much work remains to be done. Among the more challenging and potentially promising tasks is to disaggregate the "crimes against the person" output and to incorporate these variables directly into the decision problem underlying estimation. Initial work in this area seems to indicate that unit costs for clearing homocides are an order of magnitude greater than that of any other police activity.

The Hoover Institution, Stanford University

REFERENCES

Annual General Administrative Survey, prepared by Kansas City Police Department for limited circulation, 1968, 1969, 1971, 1973.

Berndt, E. R., and L. R. Christensen, "The Translog Function and the Substitution of Equipment, Structures and Labor in U.S. Manufacturing, 1929-68," Journal of Econometrics, p. 81-113, 1974.

Blackorby, C., D. Primont and R. Russell, "Dual Price and Quantity Aggregation," Journal of Economic Theory, 14, pp. 130-48, 1977.

_________, _________ and _________, "On Testing Separability Restrictions with Flexible Functional Forms," Journal of Econometrics, 1977, vol. 5, pp. 195-209.

Chapman, J. I., W. Z. Hirsch and S. Sonenblum, "Crime Prevention, The Police Production Function and Budgeting," Public Finance, Vol. XXX, no. 2, 1975, pp. 197-215.

Christensen, L. R., D. W. Jorgensen and L. J. Lau, "Conjugate Duality and the Transcendental Logarithmic Production Function," Econometrica, 39, pp. 255-56, 1971.

_________, _________ and _________, "Transcendental Logarithmic Production Frontiers," Review of Economics and Statistics, 55, pp. 28-45, 1973.

_________, _________ and _________, "Transcendental Logarithmic Utility Functions," American Economic Review, 65, pp. 367-83, 1975.

Diewert, W. E., "Aggregation Problems in the Measurement of Capital." Paper presented to the Conference on Research in Income and Wealth, National Bureau of Economic Research, Toronto, 1977.

_________, "Applications of Duality Theory," in Frontiers of Quantitative Economics, Volume II, eds. M. Intriligatorand D. A. Kendrick, North-Holland Publishing Co., 1974.

_________, "An Application of the Shephard Duality Theorem: A Generalized Leontief Production Function," Journal of Political Economy, 79 pp. 481-507, 1971.

_________, "Separability and a Generalization of the Cobb-Douglas Cost, Production and Indirect Utility Functions," Technical Report No. 86, IMSSS, Stanford University, 1973.

Ehrlich, I., "Participation in Illegitimate Activities: An Economic Analysis," in Essays in the Economics of Crime and Punishment, edited by G. Becker and W. Landes, NBER, 1975.

________, Participation in Illegitimate Activities: An Economic Analysis. Unpublished dissertation, University of Chicago, 1970.

________, "Participation in Illegitimate Activities: A Theoretical and Empirical Analysis," Journal of Political Economy, 81, pp. 521-67, 1973.

Goldman, S., and H. Uzawa, "A Note on Separability in Demand Analysis," Econometrica, 32, pp. 387-98, 1964.

Hall, R. E., "The Sepcification of Technology with Several Kinds of Output," Journal of Political Economy, 81, pp. 878-92, 1973.

Hicks, J. R., Value and Capital, Oxford: 2nd ed., Clarendon Press, 1946.

Phillips, L., and H. L. Votey, "Crime Control in California," Journal of Legal Studies, June 1975, Vol. IV, no. 2, pp. 327-49.

Popp, D. O., and F. D. Sebold, "Quasi Returns to Scale in the Provision of Police Service," Public Finance, Vol 27, no. 1, pp. 1-18, 1972.

Shephard, R. W., Cost and Production Functions, Princeton University Press, 1953.

Vandaele, Walter, "The Economics of Crime: An Econometric Investigation of Auto Theft in the United States," unpublished Ph.D. dissertation, University of Chicago, 1975.

Votey, H. L., and L. Phillips, "Police Effectiveness and the Production Function for Law Enforcement," Journal of Legal Studies, Vol 1, 1972, pp. 423-36.

Walzer, Norman, "Economies of Scale and Municipal Police Services: The Illinois Experience," Review of Economics and Statistics, Vol. 54, no. 2, 1972, pp. 431-38.

Wilson, James Q., and Barbara Boland, "The Effect of Police on Crime Rates," Urban Institute Working Paper, April 1977.