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LAW ENFORCEMENT STANDARDS PROGRAM

LIFE CYCLE COSTING TECHNIQUES
PPLICABLE TO LAW ENFORCEMENT FACILITIES

STANDARDS PROGRAM

U.S. DEPARTMENT OF JUSTICE
Law Enforcement Assistance Administration
National Institute of Law Enforcement and Criminal Justice

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LIFE CYCLE COSTING TECHNIQUES PLICABLE TO LAW ENFORCEMENT FACILITIES

prepared for the
National Institute of Law Enforcement and Criminal Justice
Law Enforcement Assistance Administration
U.S. Department of Justice

by

Robert J. Kapsch
Center for Building Technology
National Bureau of Standards

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LIFE CYCLE COSTING TECHNIQUES APPLICABLE TO LAW ENFORCEMENT FACILITIES

CONTENTS

FOREWORD

Following a Congressional mandate[1] to develop new and improved techniques, systems, and equipment to strengthen law enforcement and criminal justice, the National Institute of Law Enforcement and Criminal Justice (NILECJ) has established the Law Enforcement Standards Laboratory (LESL) at the National Bureau of Standards. LESL's function is to conduct research that will assist law enforcement and criminal justice agencies in the selection and procurement of quality equipment.

In response to priorities established by NILECJ, LESL is (1) subjecting existing equipment to laboratory testing and evaluation and (2) conducting research leading to the development of several series of documents, including national voluntary equipment standards, user guidelines, state-of-the-art surveys and other reports.

This document, LESP–RPT–0801.00, Life Cycle Costing Techniques Applicable To Law Enforcement Facilities, is a law enforcement equipment report prepared by LESL and approved and issued by NILECJ. Additional reports as well as other documents will be issued under the LESL program in the areas of protective equipment, communications equipment, security systems, weapons, emergency equipment, investigative aids, vehicles, and clothing.

Technical comments and suggestions concerning the subject matter of this report are invited from all interested parties. Comments should be addressed to the Program Manager for Standards, National Institute of Law Enforcement and Criminal Justice, Law Enforcement Assistance Administration, U.S. Department of Justice, Washington, D.C. 20531.

Lester D. Shubin, *Manager,*
Standards Program
National Institute of Law
Enforcement and Criminal Justice

[1] Section 402(b) of the Omnibus Crime Control and Safe Streets Act of 1968, as amended.

SUMMARY

Planners, architects, engineers and others engaged in the planning, design and construction of law enforcement facilities are charged with a number of decisions that will affect future resource allocations by the agency operating the constructed facility. Such future resource allocations would include the agency's being required to provide more (or fewer) personnel to operate the facility, to provide more (or less) frequent replacement of the component parts of the facility and to provide more (or less) supplies to operate the facility. Decision makers should be sensitive to the economic impact of their decisions projected over the life of the facility. The analytical tool presented in this paper for the evaluation of the economic impact of various design alternatives is the technique of life cycle costing. Through the use of this technique, the life cycle allocations by an agency for a law enforcement facility can be minimized.

LIFE CYCLE COSTING TECHNIQUES APPLICABLE TO LAW ENFORCEMENT FACILITIES

INTRODUCTION

This report is concerned with the application of techniques from building economics to the problems involved in the planning, design and construction of law enforcement facilities, including judicial or court facilities, peace officer facilities, and correctional facilities.

In the planning, design and construction of law enforcement facilities, numerous choices are made among competing alternatives. These decisions involve such radically different matters as determining the size of the planned institution, deciding upon the appropriate heating plant and choosing adequate interior finishes. These decisions involve benefits; that is, they provide amenities to the user or occupant of the facility. The benefits involve matters of safety, comfort, security, etc. In addition, these decisions involve the allocation of resources. Funds expended for penitentiaries represent funds unavailable for other purposes. In addition, building decisions involve the commitment of resources over a long period of time. More or less money expended initially for the construction of the law enforcement facility carries with it connotations of more or less resources which will have to be spent over the life of the facility. It is this latter effect of facility design and construction decision making that is the topic of this report.

The decision maker involved in the acquisition of a law enforcement facility, all else being equal, will presumably seek to minimize the expenditures for that facility while still providing an acceptable level of performance of that facility.

The report is organized into four parts. Part I, *The Basis,* explains the basic concepts involved in building economics and their applicability to the problems of law enforcement facilities. Part II, *The Formulas,* develops the mathematical formulas that are applicable to economic problem solving. Part III, *The Examples,* provides illustrations of problems and solutions involving building economics and law enforcement facilities. Finally, Part IV, *The Tables,* provides tables to aid law enforcement planning officials in applying life cycle costing techniques to the problems illustrated in this report.

This report is intended for those law enforcement officials not familiar with the techniques of discounted cash analysis or engineering economics. The bibliography contains references to additional sources of information on this subject.

I. THE BASIS

Two fundamental principles of life cycle costing are:

1. Expenditures are to be minimized over the life cycle of the facility.
2. Expenditures over the life cycle of the facility are to be calculated in accordance with the time value of money.

Together, these two principles make up the building economics technique of life cycle costing.

The first principle is self-explanatory. Decisions involving expenditures must consider not only first costs, but also future costs, usually incurred through operations, maintenance, and replacement.

The second principle, although well-known to economists, is perhaps not well-known and not widely applied in the design and construction of facilities.

Central to the second principle is the time value of money. Basically, this is the opportunity cost associated with money. That is, a dollar spent (received) today is not of the same value as a dollar spent (received) next year or the year after that. This has little to do with inflation, but instead deals with the opportunity that is available. An individual may invest a dollar in a local bank and find that it is worth $1.045 next year. Or a large corporation may invest $1000 this year and find that it is worth $1200 next year. Because the opportunities exist for investment and for a return on that investment, it is generally acknowledged that the value of money varies with time. To the successful businessman, the choice is never between alternative *A* and alternative *B*, but rather between alternatives *A*, *B* and the alternative of investing the money in some stock or bond or future market. In this way, the businessman attempts to maximize his capital return and profit.

Law enforcement facilities are obviously not profit-maximizing enterprises. Under these circumstances, is the concept of the time value of money still valid? The answer is unequivocally yes. People, firms, institutions, and even governments cannot be indifferent to the time value of money. Recently, the Department of Defense adopted a policy of recognizing the time value of money. In assessing the costs and benefits of large computer systems, Defense used the justification that expenditures represent a loss of opportunity for citizens to invest at a certain interest rate. Likewise, an expenditure of $10 million to build a new law enforcement facility is also a loss of opportunity for citizens to invest that $10 million elsewhere.

As an example of this, suppose a building manager were offered two possibilities on a boiler plant maintenance contract. The first alternative is to pay $100,000 at the end of the first year for a 2-year maintenance contract, and the second is to pay $50,000 at the end of each year for the same contract. Besides the possibility of increased control over the contractor during the second year, the second alternative is obviously superior to the first because it costs less. That is, at the end of the first year the $50,000 not given to the contractor may be invested, perhaps at 10%, to yield an additional $5000 to the institution.

Perhaps, as a further illustration of the time value of money, two types of floor material are under consideration for installation in a new law enforcement facility. Two solutions, alternatives *A* and *B*, have been identified. Both alternatives are considered adequate from a performance point of view, both are expected to last for 8 years and the only essential difference between the two is that alternative *A* is initially less expensive but more expensive to maintain than alternative *B*. This is shown below.

	Alternative A	Alternative B
Initial Cost (Year 0)	$120,000	$150,000
Maintenance Costs:		
End of Year 1	20,000	15,000
End of Year 2	20,000	15,000
End of Year 3	20,000	15,000
End of Year 4	20,000	15,000
End of Year 5	20,000	15,000
End of Year 6	20,000	15,000
End of Year 7	20,000	15,000
End of Year 8	20,000	15,000
TOTAL	$280,000	$270,000

If the initial cost alone (i.e., construction cost) were considered, then alternative *A* appears to be $30,000 less expensive than alternative *B*. If the sum of initial cost plus maintenance costs over the 8-year life of these alternatives were considered, then alternative *B* appears less expensive than alternative *A*. However, neither the comparison of initial costs nor the sums of initial costs and maintenance costs take into account the time value of money.

To compare alternatives involving different expenditures at different times, it is necessary to translate dollar amounts to an equivalent base. Costs may be converted to equivalency by use of either a present worth model or an annual cost model. The present worth model reduces all expected costs of alternative systems over an equivalent period of time to a single cost today. In the annual cost model, all costs over the life of each alternative are converted, for a given interest rate, to a series of uniform annual costs. This report describes the use of present worth models in evaluating alternative building systems.

In our example, we will translate all dollar amounts to year 0 dollars. For this example the interest rate is taken as ten percent. In translating the dollar values to base year 0 dollar amounts, the question must be asked, "How much money would have to be invested in year 0 to have $20,000?" in each of the maintenance years. Complete translations to year 0 values are shown below.

	Alternative A	Alternative B
Initial Costs (Year 0)	$120,000	$150,000
Maintenance Costs:		
Year 1 Translated	18,182	13,637
Year 2 Translated	16,528	12,396
Year 3 Translated	15,026	11,270
Year 4 Translated	13,660	10,245
Year 5 Translated	12,418	9,314
Year 6 Translated	11,290	8,468
Year 7 Translated	10,264	7,698
Year 8 Translated	9,330	6,998
TOTAL (YEAR 0) COSTS	$226,698	$230,026

From the above table, it can be seen that alternative *A*, when compared in year 0 dollars to alternative *B*, is approximately $3000 less expensive.

In the above example, it may be maintained that the shift of dollar value is not very great, the sums of money involved are very small and that one alternative may be more desirable than the other for aesthetics, convenience or other reasons. These criticisms may hold for the above example, but do not upset the principle of life cycle costing, which is extended here to planning and design considerations of new law enforcement facilities of both substantial cost and of long life spans.

In summary, the analysis of different alternatives with different expenditures over time, when considering the time value of money, is more complicated than simply summing future expenditures.

2. THE FORMULAS

From the example in the preceding part, it may have been implied that the determination of present values is made by trial and error. Of course, this is not the case. Rather, there are appropriate formulas that can be utilized.

Suppose we invested a sum of money, *P*, at an annual interest rate, *i*, and wanted to know the total amount, *F*, we would have at the end of the first year; at the end of the second year, etc. We could proceed as follows:

Year	*Amount of Money*
0	P
1	$F_1 = P(1+i)$
2	$F_2 = P(1+i)(1+i)$
3	$F_3 = P(1+i)(1+i)(1+i)$
—	
N	$F = P(1+i)^N$

$$F = P(1+i)^N \qquad \text{(Equation 1)}$$

or $$P = F\left[\frac{1}{(1+i)^N}\right] \qquad \text{(Equation 2)}$$

To illustrate the above, if \$50,000 were invested in year 0 at 10 percent interest, what amount would be available in year 2?

$$F_2 = P(1+i)^N$$
$$F_2 = \$50{,}000\ (1+.10)^2$$
$$F_2 = \$50{,}000\ (1.21)$$
$$F_2 = \$60{,}500$$

Suppose we intended to invest a sum of money, A, at the end of the first year and an additional amount, A, at the end of each subsequent year, at i percent interest, and wanted to know how much we would have at the end of year 1 (F_1), 2 (F_2), 3 (F_3), etc. We would proceed as follows:

Year	*Amount of Money*
1	$F_1=A$
2	$F_2=A+A(1+i)$
3	$F_3=A+A(1+i)+A(1+i)(1+i)$
4	$F_4=A+A(1+i)+A(1+i)(1+i)+A(1+i)(1+i)(1+i)$
N	$F_N=A+A(1+i)^1+A(1+i)^2+A(1+i)^3+\ldots A(1+i)^{N-2}+A(1+i)^{N-1}$

or

$$F_N=A[1+(1+i)^1+(1+i)^2+\ldots(1+i)^{N-1}]$$

Both sides of this equation may be multiplied by $(1+i)$ producing the new equation:

$$(1+i)F_N=A[(1+i)+(1+i)^2+(1+i)^3+\ldots(1+i)^N]$$

The first equation can be subtracted from the second to produce:

$$iF_N=A[(1+i)^N-1]$$

or

$$F_N=A\left[\frac{(1+i)^N-1}{i}\right] \qquad \text{(Equation 3)}$$

or

$$A=F\left[\frac{i}{(1+i)^N-1}\right] \qquad \text{(Equation 4)}$$

To illustrate the use of the above equations, suppose \$25.000 were invested at the end of each year for 5 consecutive years at the annual interest rate of 8 percent. What would the cumulative amount be at the end of the fifth year?

$$F_5=A\left[\frac{(1+i)^N-1}{i}\right] \qquad \text{(Equation 3)}$$

$$F_5=\$25{,}000\left[\frac{(1+.08)^5-1}{0.08}\right]$$

$$F_5=\$25{,}000\left[\frac{(1.46933)-1}{0.08}\right]$$

$$F_5 = \$25{,}000[5.8667]$$

$$F_5 = \$146{,}668$$

Equations 1 and 2 indicate the relationship between F, a future sum, and P, a present sum. Equations 3 and 4 indicate the relationship between F, a future sum, and A, a uniform series of investments over N periods. This leaves the relationship between P, a present sum, and A, a uniform series, to be derived for our use.

We have:

$$A = F\left[\frac{i}{(1-i)^N - 1}\right] \qquad \text{(Equation 4)}$$

We also know:

$$F = P(1+i)^N \qquad \text{(Equation 1)}$$

Substituting:

$$A = P(1+i)^N\left[\frac{i}{(1+i)^N - 1}\right]$$

Or:

$$A = P\left[\frac{i(1+i)^N}{(1+i)^N - 1}\right] \qquad \text{(Equation 5)}$$

Similarly:

$$P = A\left[\frac{(1+i)^N - 1}{i(1+i)^N}\right] \qquad \text{Equation 6}$$

To illustrate the use of the above equations, what is the present worth, P, of \$7500 a year, A, invested each year for the next 7 years at 5% interest, i?

$$P = A\left[\frac{(1+i)^N - 1}{i(1+i)^N}\right] \qquad \text{(Equation 6)}$$

$$P = \$7500\left[\frac{(1+.05)^7 - 1}{.05(1+.05)^7}\right]$$

$$P = \$7500\left[\frac{(1.40710) - 1}{.05(1.40710)}\right]$$

$$P = \$7500\left[\left(\frac{.40710}{.070355}\right)\right]$$

$$P = \$7500\,(5.7864)$$

$$P = \$43{,}398$$

To summarize:

Given P; to Find F	Equation 1	$F = P(1+i)^N$
Given F; to Find P	Equation 2	$P = F\left[\frac{1}{(1+i)^N}\right]$

Given A; to Find F	Equation 3	$F=A\left[\frac{(1+i)^N-1}{i}\right]$
Given F; to Find A	Equation 4	$A=F\left[\frac{i}{(1+i)^N-1}\right]$
Given P; to Find A	Equation 5	$A=P\left[\frac{i(1+i)^N}{(1+i)^N-1}\right]$
Given A; to Find P	Equation 6	$P=A\,\frac{(1+i)^N-1}{i(1+i)^N}$

Where:
P = Present sum of money.
F = Future sum of money that is equivalent to P at the end of N periods of time at an interest of i.
i = Interest rate.
N = Number of interest periods.
A = End-of-period payment (or receipt) in a uniform series of payments (or receipts) over N periods at i interest rate.

Finally, we can identify these formulas by the following standard nomenclature and shorthand notations, originally developed by the Engineering Economy Division of the American Society for Engineering Education.*

STANDARD NOMENCLATURE AND NOTATION

Use When	Algebraic Form	Standard Nomenclature	Standard Notation	Equation #
Given P; to find F	$F=P(1+i)^N$	Compound Amount Factor (Single Payment)	$(F/P, i\%, N)$	1
Given F; to find P	$P=F\left[\frac{1}{(1+i)^N}\right]$	Present Worth Factor (Single Payment)	$(P/F, i\%, N)$	2
Given F; to find A	$A=F\left[\frac{i}{(1+i)^N-1}\right]$	Sinking Fund Factor	$(A/F, i\%, N)$	4
Given P; to find A	$A=P\left[\frac{i(1+i)^N}{(1+i)^N-1}\right]$	Capital Recovery Factor	$(A/P, i\%, N)$	5
Given A; to find F	$F=A\left[\frac{(1+i)^N-1}{i}\right]$	Compound Amount Factor (Uniform Series)	$(F/A, i\%, N)$	3
Given A; to find P	$P=A\left[\frac{(1+i)^N-1}{i(1+i)^N}\right]$	Present Worth Factor (Uniform Series)	$(P/A, i\%, N)$	6

*Prepared by the Committee on Standardization of Engineering Economy Notation, "Manual of Standard Notation for Engineering Economy Parameters and Interest Factors," Engineering Economy Division, American Society for Engineering Education. Updated. Copies of this report are available from Dr. Arthur Lesser, Jr., Editor, The Engineering Economist, Stevens Institute of Technology, Hoboken, New Jersey 07030.

3. THE EXAMPLES

Life cycle cost analysis is a technique that can be applied at any level of design and construction of a law enforcement facility. To demonstrate this, three examples are provided as follows: Example One will illustrate this technique in the selection of a building material; Example Two will deal with a building subsystem; and, Example Three will deal with the macro, or overview, level of facility alternatives assessment.

Example One

This first example illustrates the use of life cycle cost analysis at the lowest level of decision-making encountered in the design and construction of law enforcement facilities; the selection of building materials. In particular, this example illustrates the use of life cycle cost analysis in the decision between two competing floor coverings; floor covering A and floor covering B. This could involve a decision between asphalt tile and vinyl asbestos tile, or between an expensive resilient tile and an inexpensive indoor-outdoor carpeting. Typically, one alternative will have a lower initial cost and the other alternative will have a longer life or require less maintenance. It is assumed that either alternative A or alternative B will meet all of the other performance requirements. In other words, the differentiation between floor covering A and floor covering B can be made solely on the basis of cost.

For this illustration, assume that a general purpose office area is to be covered with either floor covering A or B. The area involved is 10,000 square feet (929 square meters). The initial costs of these alterations are as follows:

Initial Cost of A = I.C.(A) = $0.42 per square foot ($4.52 per square meter)
Initial Cost of B = I.C.(B) = $0.58 per square foot ($6.18 per square meter).

Both costs represent installed cost (labor and material) and have been appropriately estimated to reflect the size and location of the building involved.

Alternative A is judged to have a shorter life than B. Based on government reports, it is estimated that alternative A must be replaced every 5 years and B must be replaced every 7 years. The estimated life of the building is 35 years.

Exact future costs of the replacement of A and B are not known, of course. However, it is known that since World War II, the installed cost of A has shown a 2 percent per year increase while B has shown a 3 percent per year increase. It is expected that these general trends will continue.

Finally, maintenance on alternative B is less than that of A. For the first year, it is estimated that maintenance for the alternatives are as follows:

Maintenance Cost of A = M.C.(A) = $0.15 per square foot per year ($1.61 per square meter per year)
Maintenance Cost of B = M.C.(B) = $0.14 per square foot per year ($1.50 per square meter per year).

It is expected that these costs will continue to grow at the rate of 5 percent per year for the life of the building.

The problem is: Which alternative is less expensive over the life of the building?

Generally, two equations can be written.

$$\text{L.C.C.}(A) = \text{I.C.}(A) + \text{R.C.}(A) + \text{M.C.}(A)$$
$$\text{L.C.C.}(B) = \text{I.C.}(B) + \text{R.C.}(B) + \text{M.C.}(B)$$

where:

L.C.C. = Life cycle cost.
I.C. = Initial cost.
R.C. = Replacement cost.
M.C. = Maintenance cost.

The above equations are based on the assumption that all costs are to be comparable; i.e., they are to be translated to the same base year.

To develop these general equations further, we will expand each term as it appears on the right hand side of the equations.

Initial Cost (I.C.) Initial costs are the only ones already in terms of present value; that is, initial costs do not require translation.

Therefore:

$$\text{Initial Cost of } A = \text{I.C.}(A) = \$0.42 \times 10{,}000 = \$4200$$
$$\text{Initial Cost of } B = \text{I.C.}(B) = \$0.58 \times 10{,}000 = \$5800$$

Replacement Cost (R.C.) Assuming that the beneficial occupancy of this facility occurs in 1973, we can anticipate the following replacement schedules:

Replacement of A: 1978, 1983, 1988, 1993, 1998, and 2003

Replacement of B: 1980, 1987, 1994, and 2001

The cost of these replacements can be estimated by projecting the initial costs at a 2 percent increase per year (Alternative A) and a 3 percent per year (Alternative B). Utilizing Equation 1, $F = P(1+i)^N$, the following costs are calculated:

Alternative A:

Cost of Replacement in year	
1978	$= \$4200 \text{ X } (1.02)^5 = \$4200 \text{ X } (1.104) = \$4637$
1983	$= \$4200 \text{ X } (1.02)^{10} = \$4200 \text{ X } (1.219) = \$5120$
1988	$= \$4200 \text{ X } (1.02)^{15} = \$4200 \text{ X } (1.346) = \$5653$
1993	$= \$4200 \text{ X } (1.02)^{20} = \$4200 \text{ X } (1.486) = \$6241$
1998	$= \$4200 \text{ X } (1.02)^{25} = \$4200 \text{ X } (1.641) = \$6892$
2003	$= \$4200 \text{ X } (1.02)^{30} = \$4200 \text{ X } (1.811) = \$7606$

Rather than calculate quantities such as $(1.02)^{30}$, these quantities can be taken from Table 1, in the following part (Part IV). Cost of replacement for alternative B can similarly be calculated:

Alternative B:

Cost of Replacement in year	
1980	$= \$5800 \text{ X } (1.03)^7 = \$5800 \text{ X } (1.230) = \$7134$
1987	$= \$5800 \text{ X } (1.03)^{14} = \$5800 \text{ X } (1.513) = \$8775$
1994	$= \$5800 \text{ X } (1.03)^{21} = \$5800 \text{ X } (1.860) = \$10{,}788$
2001	$= \$5800 \text{ X } (1.03)^{28} = \$5800 \text{ X } (2.288) = \$13{,}270$

The above dollar figures represent estimated future cash outlays but are not comparable, since the time value of money has not been taken into consideration. By applying the time value of money, we are, in effect, translating future sums into present terms according to some interest rate, i. This can be done by means of Equation 2,

$$P = F\left[\frac{1}{(1+i)^N}\right]$$

The interest rate to be used will be 10 percent on the theory that private firms might receive 10 percent if they were not deprived of the opportunity by taxes; i.e., such taxes as those needed to construct law enforcement facilities. The present value of replacement can be calculated as follows:

Alternative A:

Present Value of:

1978 Replacement $= \$4637 \left[\frac{1}{(1+.10)^5}\right] = \$4637\ (.6209) = \$2879$

1983 Replacement $= \$5120 \left[\frac{1}{(1+.10)^{10}}\right] = \$5120\ (.3855) = \$1974$

1988 Replacement $= \$5653 \left[\frac{1}{(1+.10)^{15}}\right] = \$5653\ (.2394) = \$1353$

1993 Replacement $= \$6241 \left[\frac{1}{(1+.10)^{20}}\right] = \$6241\ (.1486) = \$\ 927$

1998 Replacement $= \$6892 \left[\frac{1}{(1+.10)^{25}}\right] = \$6892\ (.0923) = \$\ 636$

2003 Replacement $= \$7606 \left[\frac{1}{(1+.10)^{30}}\right] = \$7606\ (.0573) = \underline{\$\ 436}$

TOTAL COST OF REPLACEMENTS (1973 dollars) $8205

Therefore R.C. (A)–$8205
Similarly for alternative B:

Alternative B:

Present Value of:

1980 Replacement $= \$\ 7134 \left[\frac{1}{(1+.10)^7}\right] = \$\ 7134(0.5132) = \$3661$

1987 Replacement $= \$\ 8{,}775 \left[\frac{1}{(1+.10)^{14}}\right] = \$\ 8{,}775(0.2633) = \$2310$

1994 Replacement $= \$10{,}788 \left[\frac{1}{(1+.10)^{21}}\right] = \$10{,}788(0.1351) = \$1457$

2001 Replacement $= \$13{,}270 \left[\frac{1}{(1+.10)^{28}}\right] = \$13{,}270(0.0693) = \underline{\$\ 920}$

TOTAL COST OF REPLACEMENTS (1973 dollars= $8348

Therefore R.C.(B)–$8348

Algebraically, the above operations can be written:

$$R.C. = I.C.(1+i_x)^m \left[\frac{1}{(1+i_o)^m}\right] + I.C.(1+i_x)^{2m}\left[\frac{1}{(1+i_o)^{2m}}\right] +$$

$$I.C.(1+i_x)^{3m} \left[\frac{1}{(1+i_o)^{3m}}\right] + \ldots I.C.(1+i_x)^{L-m} \left[\frac{1}{(1+i_o)^{L-m}}\right]$$

where:

R.C. = Replacement cost (in terms of 1973 dollars).
I.C. = Initial cost (in terms of 1973 dollars).

i_x = Expected percentage yearly cost increase, expressed as a decimal.
i_o = Opportunity cost.
m = Expected life of the floor covering, expressed in years.
L = Life of the building, expressed in years.

Maintenance Cost. The nominal initial maintenance costs can be calculated as follows:

$$\text{M.C.}(A) = 10{,}000 \times \$0.15 \text{ per square foot per year} = \$1500$$
$$\text{M.C.}(B) = 10{,}000 \times \$0.14 \text{ per square foot per year} = \$1400$$

Present value costs for the 35 years of maintenance must be calculated in a manner similar to that shown for replacement cost. This is shown in Table E–1 on page 11.

Using the standard nomenclature, the operation performed in Table E–1 can be written:

$$\begin{aligned}\text{Total M.C.} = {} & \text{M.C.}(F/P, i_x, 1)(P/F, i_o, 1) \\ & + \text{M.C.}(F/P, i_x, 2)(P/F, i_o, 2) \\ & + \ldots \text{M.C.}(F/P, i_x, L)(P/F, i_o, L)\end{aligned}$$

Life Cycle Cost. Total life cycle cost can then be arrived at by summing initial cost, replacement cost and maintenance cost, all of which are now expressed in terms of 1973 dollars.

Life Cycle Cost (A) = L.C.C.(A) = I.C.(A) + R.C.(A) + M.C.(A)
L.C.C.(A) = \$4200 + \$8205 + \$25,321
L.C.C.(A) = \$37,726

Similarly,

Life Cycle Cost (B) = L.C.C.(B) = I.C.(B) + R.C.(B) + M.C.(B)
L.C.C.(B) = \$5800 + \$8348 + \$23,630
L.C.C.(B) = \$37,778

So, despite the fact that alternative B is almost 40 percent more expensive than alternative A initially, the life cycle costs of the two alternatives are approximately the same. The choice of one over the other can be based on considerations other than cost.

In this example, all future projections were assumed. In a real problem the determination of future costs and cost trends is difficult, especially where trend data is not available. Because of the difficulty of forecasting the future, the usual procedure is to develop a computer model, based on the formulas shown above, and to try different sets of values for the variables. In our example, we would try various reasonable values of i_o, i_x, L, m, etc. to see how these variations affect the final outcome. This procedure is called sensitivity analysis. The exact dollar value of either alternative A or B is not as important here as the dollar value of A relative to B. If reasonable changes in the variables still produce the same outcome, then the design decision remains the same.

Example Two

The second example illustrates the use of life cycle cost analysis at the building assembly, or building subsystem level of decision making. In particular, this example deals with the selection of an appropriate central heating facility for a new state prison complex. We will assume that from the many possibilities available, all but two have already been eliminated.

Of these, alternative X is more expensive initially and utilizes a more expensive fuel. Alternative Y is less expensive but the price of its fuel, while presently low, has been rising sharply in the past 10 years, and this trend can be expected to continue.

Quantitatively, the decision between alternative X and Y is as follows:

Alternative	Initial Cost	Annual Cost of Fuel	Percent Increase Cost of Fuel
X	$320,000	$55,000/year	3 percent/year
Y	$280,000	$45,000/year	8 percent/year

For the purposes of this illustration, it is assumed that maintenance costs, replacement costs and life spans are equal. The central question, is "What life of this structure will justify alternative *X* over alternative *Y*?" That is, how long must the plant be in

TABLE E–1

Year	$i_x = 5\%$ $(F/P, i_x, n)$		$i_o = 10\%$ $(P/F, i_o, n)$		Product	Alternative A	Alternative B
						Product × $1500	Product × $1400
1974	1.050	×	0.9091	=	0.9546	$1432	$1336
1975	1.103	×	0.8264	=	0.9115	1367	1276
1976	1.158	×	0.7513	=	0.8700	1305	1218
1977	1.216	×	0.6830	=	0.8305	1246	1163
1978	1.276	×	0.6209	=	0.7923	1188	1109
1979	1.340	×	0.5645	=	0.7564	1135	1059
1980	1.407	×	0.5132	=	0.7221	1083	1011
1981	1.477	×	0.4665	=	0.6890	1034	965
1982	1.551	×	0.4241	=	0.6578	987	921
1983	1.629	×	0.3855	=	0.6280	942	879
1984	1.710	×	0.3505	=	0.5994	899	839
1985	1.796	×	0.3186	=	0.5722	858	801
1986	1.886	×	0.2897	=	0.5464	820	765
1987	1.980	×	0.2633	=	0.5213	782	730
1988	2.079	×	0.2394	=	0.4977	747	697
1989	2.183	×	0.2176	=	0.4750	713	665
1990	2.292	×	0.1978	=	0.4534	680	635
1991	2.407	×	0.1799	=	0.4330	650	606
1992	2.527	×	0.1635	=	0.4132	620	578
1993	2.653	×	0.1486	=	0.3942	591	552
1994	2.786	×	0.1351	=	0.3764	565	527
1995	2.925	×	0.1228	=	0.3592	539	503
1996	3.072	×	0.1117	=	0.3431	515	480
1997	3.225	×	0.1015	=	0.3273	491	458
1998	3.386	×	0.0923	=	0.3125	469	438
1999	3.556	×	0.0839	=	0.2983	447	418
2000	3.733	×	0.0763	=	0.2848	427	399
2001	3.920	×	0.0693	=	0.2717	408	380
2002	4.116	×	0.0630	=	0.2593	389	363
2003	4.322	×	0.0573	=	0.2477	372	347
2004	4.538	×	0.0521	=	0.2364	355	331
2005	4.765	×	0.0471	=	0.2259	339	316
2006	5.003	×	0.0431	=	0.2156	323	302
2007	5.253	×	0.0391	=	0.2054	308	288
2008	5.516	×	0.0356	=	0.1964	295	275
MAINTENANCE COST				=	TOTAL	$25,321	$23,630

operation until fuel savings from alternative Y offset the higher initial cost of alternative X? Assume the opportunity cost of money is 5 percent (i_o).

Two equations can be written:

$$\text{Life Cycle Cost of } X = \text{L.C.C. } (X) = \text{I.C.} + \$55{,}000(1+i_x)^1\left[\frac{1}{(1+i_o)^1}\right]$$

$$+ \$55{,}000(1+i_x)^2\left[\frac{1}{(1+i_o)^2}\right] + \cdots \$55{,}000(1+i_x)^L\left[\frac{1}{(1+i_o)^L}\right]$$

$$\text{Life Cycle Cost of } Y = \text{L. C. C. } (Y) = \text{I. C.}$$

$$+ \$45{,}000(1+i_Y)^1\left[\frac{1}{(1+i_o)^1}\right] + \$45{,}000(1+i_Y)^2\left[\frac{1}{(1+i_o)^2}\right]$$

$$+ \ldots \$45{,}000(1+i_Y)^L\left[\frac{1}{(1+i_o)^L}\right]$$

Where:

I. C. = Initial cost.

i_x = Expected percentage yearly cost increase of fuel of alternate X, expressed as a decimal.

i_Y = Expected percentage yearly cost increase of fuel of alternate Y, expressed as a decimal.

i_o = Opportunity cost.

L = Life of the plant.

We can set L. C. C. (X) equal to L. C. C. (Y) and solve for L, to determine at what point in time alternative X will begin to be less expensive than alternative Y. The computed values are listed in Table E–2 on page 13.

From Table E–2, it can be seen that the fuel associated with alternative Y becomes more expensive than the fuel associated with alternative X somewhere between the fourth and fifth year, as measured in terms of the present values of these future projected cash outlays. In terms of total life cycle cost, alternative Y becomes more expensive than alternative X between the tenth and eleventh year. Since law enforcement facilities are typically in use for periods greatly exceeding the 10-to-11 year break-even point, alternative X would be deemed the more economical choice from the life cycle cost viewpoint.

Example Three

The third example deals with an overview of the facility acquisition process. Specifically, this example deals with the question of buying versus leasing and the application of life cycle cost analysis to aid in this decision.

Assume that an experimental half-way house program is to be established for 5 years by the State. This program requires a 4,000 square foot (370 square meter) facility in the immediate vicinity of a medium size city. A suitable building is commercially available at $9600 per year for 5 years. Instead of leasing this facility, the State could elect to build its own facility at an initial cost of $120,000 ($30 per square foot, including land) and an operating cost of $900 per year. If the program is discontinued at the end of the 5-year period, it is expected that sale of the building would result in a revenue of $140,000. Is it less expensive for the State to lease or buy? Assume that the State, like the Department of Defense, uses a discount rate of 10 percent (i = 10 percent).

TABLE E–2

L.C.C.(X) (All dollar figures in terms of year 0 dollars) $i_x=3\%$ $i_o=5\%$ I.C.=$320,000					
N	$(1+i_x)^N$	$\frac{1}{(1+i_o)^N}$	Product	Times $55,000 (Fuel Cost)	L.C.C.(X) Subtotal
1	1.030	0.9524	0.9810	53,955	373,955
2	1.061	0.9070	0.9623	52,927	426,882
3	1.093	0.8638	0.9441	51,926	478,808
4	1.126	0.8227	0.9264	50,952	529,760
5	1.159	0.7835	0.9081	49,946*	579,706
6	1.194	0.7462	0.8910	49,005	628,711
7	1.230	0.7107	0.8742	48,081	676,792
8	1.267	0.6768	0.8575	47,163	723,955
9	1.305	0.6446	0.8412	46,266	770,221
10	1.344	0.6139	0.8251	45,381	815,602
11	1.384	0.5847	0.8092	44,506	860,108*
12	1.426	0.5568	0.7940	43,670	903,778
13	1.469	0.5303	0.7790	42,845	946,623
14	1.513	0.5051	0.7642	42,031	988,654
15	1.558	0.4810	0.7494	41,217	1,029,871
16	1.605	0.4581	0.7353	40,442	1,070,313
17	1.653	0.4363	0.7212	39,666	1,109,979
18	1.702	0.4155	0.7072	38,896	1,148,875
19	1.754	0.3957	0.6941	38,176	1,187,051

L.C.C.(Y) (All dollar figures in terms of year 0 dollars) $i_y=8\%$ $i_o=5\%$ I.C.=$280,000					
N	$(1+i_y)^N$	$\frac{1}{(1+i_o)^N}$	Product	Times $45,000 (Fuel Cost)	L.C.C.(Y) Subtotal
1	1.080	0.9524	1.0286	46,287	326,287
2	1.166	0.9070	1.0576	47,592	373,879
3	1.260	0.8638	1.0884	48,978	422,857
4	1.360	0.8227	1.1189	50,351	473,208
5	1.469	0.7835	1.1510	51,795*	525,003
6	1.587	0.7462	1.1842	53,289	578,292
7	1.714	0.7107	1.2181	54,815	633,107
8	1.851	0.6768	1.2528	56,376	689,483
9	1.999	0.6446	1.2886	57,987	747,470
10	2.159	0.6139	1.3254	59,643	807,113
11	2.332	0.5847	1.3635	61,358	868,471*
12	2.518	0.5568	1.4020	63,090	931,561
13	2.720	0.5303	1.4424	64,908	996,469
14	2.937	0.5051	1.4835	66,758	1,063,227
15	3.172	0.4810	1.5257	68,657	1,131,884
16	3.426	0.4581	1.5695	70,628	1,202,512
17	3.700	0.4363	1.6143	72,644	1,275,156
18	3.996	0.4155	1.6603	74,714	1,349,870
19	4.316	0.3957	1.7078	76,851	1,426,721

Total Cost of Lease. The cost of the lease is \$9600 per year. This can be reduced to present value by the following formula:

$$P_L = \$9600 \left[\frac{(1+i)^5 - 1}{i(1+i)^5} \right] \quad \text{Where:} \quad i = 10\%$$

$$P_L = \text{Total Cost of Lease}$$

$$P_L = \$9600 \left[\frac{0.61051}{0.16105} \right] = \$9600\ (3.791) = \$36{,}394$$

$$P_L = \$36{,}394$$

Total Cost of Buying. The cost of buying the necessary building can be reduced to present value by the following formula:

$$P_B = \text{Initial Cost} + \text{Present Value of Operations Cost} - \text{Present Value of Salvage Revenue}$$

This can be written:

$$P_B = \$120{,}000 + \$900 \left[\frac{(1+i)^5 - 1}{i(1+i)^5} \right] - \$140{,}000 \left[\frac{1}{(1+i)^5} \right]$$

where i is 10 percent and P_B is the total cost of buying the facility.

$$P_B = \$120{,}000 + \$900\ (3.791) - \$140{,}000\ (0.6209)$$
$$P_B = \$120{,}000 + \$3412 - \$86{,}926$$
$$P_B = \$36{,}486$$

As in Example One, the decision between lease and buy must depend upon other factors when the total cost figures are this close.

4. THE TABLES

Each of the following six tables corresponds to one of the equations developed in Part II, The Formulas. The tables allow the user to avoid a great deal of calculation in the application of the formulas.

Example of Use of the Tables

Assume that it is desired to determine the future value (F) of \$15,000 ($P = \$15{,}000$) invested for 13 years ($N = 13$) at an annual interest rate of 8 percent ($i = 8\%$). This can be calculated through the use of equation 1:

$$F = P(1+i)^N$$

However, to avoid the calculation (1+.08) raised to the thirteenth power, its value can be looked up in table 1 and found to equal 2.720. To calculate F, the future sum of money, this factor is multiplied by P, the present sum:

$$F = \$15{,}000\ (2.720)$$
$$F = \$40{,}800$$

THE TABLES

		Standard Notation	Algebraic Formula
TABLE 1	Compound Amount Factor (Single Payment)	$(F/P, i\%, N)$	$F=P(1+i)^N$
TABLE 2	Present Worth Factor (Single Payment)	$(P/F, i\%, N)$	$P=F\left[\frac{1}{(1+i)^N}\right]$
TABLE 3	Sinking Fund Factor	$(A/F, i\%, N)$	$A=F\left[\frac{i}{(1+i)^N-1}\right]$
TABLE 4	Capital Recovery Factor	$(A/P, i\%, N)$	$A=P\left[\frac{i(1+i)^N}{(1+i)^N-1}\right]$
TABLE 5	Compound Amount Factor (Uniform Series)	$(F/A, i\%, N)$	$F=A\left[\frac{(1+i)^N-1}{i}\right]$
TABLE 6	Present Worth Factor (Uniform Series)	$(P/A, i\%, N)$	$P=A\left[\frac{(1+i)^N-1}{i(1+i)^N}\right]$

WHERE: P = Present sum of money.
F = Future sum of money that is equivalent to P at the end of N periods of time at an interest i.
i = Interest rate.
N = Number of interest periods.
A = End-of-period payment or receipt in a uniform series of payments or receipts over N periods at i interest rate.

TABLE 1. *Compound Amount Factor (Single Factor); Given P, to Find F*

N	$i=1\%$	$i=2\%$	$i=3\%$	$i=4\%$	$i=5\%$	$i=8\%$	$i=10\%$	$i=12\%$	$i=15\%$	$i=20\%$	N
1	1.010	1.020	1.030	1.040	1.050	1.080	1.100	1.120	1.150	1.200	1
2	1.020	1.040	1.061	1.082	1.103	1.166	1.210	1.254	1.322	1.440	2
3	1.030	1.061	1.093	1.125	1.158	1.260	1.331	1.405	1.521	1.728	3
4	1.041	1.082	1.126	1.170	1.216	1.360	1.464	1.574	1.749	2.074	4
5	1.051	1.104	1.159	1.217	1.276	1.469	1.611	1.762	2.011	2.488	5
6	1.062	1.126	1.194	1.265	1.340	1.587	1.772	1.974	2.313	2.986	6
7	1.072	1.149	1.230	1.316	1.407	1.714	1.949	2.211	2.660	3.583	7
8	1.083	1.172	1.267	1.369	1.477	1.851	2.144	2.476	3.059	4.300	8
9	1.094	1.195	1.305	1.423	1.551	1.999	2.358	2.773	3.518	5.160	9
10	1.105	1.219	1.344	1.480	1.629	2.159	2.594	3.106	4.046	6.192	10
11	1.116	1.243	1.384	1.539	1.710	2.332	2.853	3.479	4.652	7.430	11
12	1.127	1.268	1.426	1.601	1.796	2.518	3.138	3.896	5.350	8.916	12
13	1.138	1.294	1.469	1.665	1.886	2.720	3.452	4.363	6.153	10.699	13
14	1.149	1.319	1.513	1.732	1.980	2.937	3.797	4.887	7.076	12.839	14
15	1.161	1.346	1.558	1.801	2.079	3.172	4.177	5.474	8.137	15.407	15
20	1.220	1.486	1.806	2.191	2.653	4.661	6.727	9.646	16.367	38.338	20
25	1.282	1.641	2.094	2.666	3.386	6.848	10.835	17.000	32.919	95.396	25
30	1.348	1.811	2.427	3.243	4.322	10.063	17.449	29.960	66.212	237.376	30
35	1.417	2.000	2.814	3.946	5.516	14.785	28.102	52.800	133.175	590.668	35
40	1.489	2.208	3.262	4.801	7.040	21.725	45.259	93.051	267.862	1469.771	40
45	1.565	2.438	3.782	5.841	8.985	31.920	72.890	163.988	538.769	3657.260	45
50	1.645	2.692	4.384	7.107	11.467	46.902	117.391	289.002	1083.652	9100.427	50
60	1.817	3.281	5.892	10.520	18.679	101.257	304.482				60
75	2.109	4.416	9.179	18.945	38.833	321.205	1271.895				75
100	2.705	7.245	19.219	50.505	131.501	2199.761					100

TABLE 2. *Present Worth Factor (Single Payment); Given* F, *to find* P

N	i=1%	i=2%	i=3%	i=4%	i=5%	i=8%	i=10%	i=12%	i=15%	i=20%	N
1	0.9901	0.9804	0.9709	0.9615	0.9524	0.9259	0.9091	0.8929	0.8696	0.8333	1
2	0.9803	0.9612	0.9426	0.9246	0.9070	0.8573	0.8264	0.7972	0.7561	0.6944	2
3	0.9706	0.9423	0.9151	0.8890	0.8638	0.7938	0.7513	0.7118	0.6575	0.5787	3
4	0.9610	0.9238	0.8885	0.8548	0.8227	0.7350	0.6830	0.6355	0.5718	0.4823	4
5	0.9515	0.9057	0.8626	0.8219	0.7835	0.6806	0.6209	0.5674	0.4972	0.4019	5
6	0.9420	0.8880	0.8375	0.7903	0.7462	0.6302	0.5645	0.5066	0.4323	0.3349	6
7	0.9327	0.8706	0.8131	0.7599	0.7107	0.5835	0.5132	0.4523	0.3759	0.2791	7
8	0.9235	0.8535	0.7894	0.7307	0.6768	0.5403	0.4665	0.4039	0.3269	0.2326	8
9	0.9143	0.8368	0.7664	0.7026	0.6446	0.5002	0.4241	0.3606	0.2843	0.1938	9
10	0.9053	0.8203	0.7441	0.6756	0.6139	0.4632	0.3855	0.3220	0.2472	0.1615	10
11	0.8963	0.8043	0.7224	0.6496	0.5847	0.4289	0.3505	0.2875	0.2149	0.1346	11
12	0.8874	0.7885	0.7014	0.6246	0.5568	0.3971	0.3186	0.2567	0.1869	0.1122	12
13	0.8787	0.7730	0.6810	0.6006	0.5303	0.3677	0.2897	0.2292	0.1625	0.0935	13
14	0.8700	0.7579	0.6611	0.5775	0.5051	0.3405	0.2633	0.2046	0.1413	0.0779	14
15	0.8613	0.7430	0.6419	0.5553	0.4810	0.3152	0.2394	0.1827	0.1229	0.0649	15
20	0.8195	0.6730	0.5537	0.4564	0.3769	0.2145	0.1486	0.1037	0.0611	0.0261	20
25	0.7798	0.6095	0.4776	0.3751	0.2953	0.1460	0.0923	0.0588	0.0304	0.0105	25
30	0.7419	0.5521	0.4120	0.3083	0.2314	0.0994	0.0573	0.0334	0.0151	0.0042	30
35	0.7059	0.5000	0.3554	0.2534	0.1813	0.0676	0.0356	0.0189	0.0075	0.0017	35
40	0.6717	0.4529	0.3066	0.2083	0.1420	0.0460	0.0221	0.0107	0.0037	0.0007	40
45	0.6391	0.4102	0.2644	0.1712	0.1113	0.0313	0.0137	0.0061	0.0019	0.0003	45
50	0.6080	0.3715	0.2281	0.1407	0.0872	0.0213	0.0085	0.0035	0.0009	0.0001	50
60	0.5504	0.3048	0.1697	0.0951	0.0535	0.0099	0.0033	0.0011	0.0002		60
75	0.4741	0.2265	0.1089	0.0528	0.0258	0.0031	0.0008	0.0002			75
100	0.3697	0.1380	0.0520	0.0198	0.0076	0.0005	0.0001				100

TABLE 3. *Sinking Fund Factor; Given F, to Find A*

N	i=1%	i=2%	i=3%	i=4%	i=5%	i=8%	i=10%	i=12%	i=15%	i=20%	N
1	1.00000	1.00000	1.00000	1.00000	1.00000	1.00000	1.00000	1.00000	1.00000	1.00000	1
2	0.49751	0.49505	0.49261	0.49020	0.48780	0.48077	0.47619	0.47170	0.46512	0.45455	2
3	0.33002	0.32675	0.32353	0.32035	0.31721	0.30803	0.30211	0.29635	0.28798	0.27473	3
4	0.24638	0.24262	0.23903	0.23549	0.23201	0.22192	0.21547	0.20923	0.20027	0.18629	4
5	0.19604	0.19216	0.18835	0.18463	0.18097	0.17046	0.16380	0.15741	0.14832	0.13438	5
6	0.16255	0.15853	0.15460	0.15076	0.14702	0.13632	0.12961	0.12323	0.11424	0.10071	6
7	0.13863	0.13451	0.13051	0.12661	0.12282	0.11207	0.10541	0.09912	0.09036	0.07742	7
8	0.12069	0.11651	0.11246	0.10853	0.10472	0.09401	0.08744	0.08130	0.07285	0.06061	8
9	0.10674	0.10252	0.09843	0.09449	0.09069	0.08008	0.07364	0.06768	0.05957	0.04808	9
10	0.09558	0.09133	0.08723	0.08329	0.07950	0.06903	0.06275	0.05698	0.04925	0.03852	10
11	0.08645	0.08218	0.07808	0.07415	0.07039	0.06008	0.05396	0.04842	0.04107	0.03110	11
12	0.07885	0.07456	0.07046	0.06655	0.06283	0.05270	0.04676	0.04144	0.03448	0.02527	12
13	0.07241	0.06812	0.06403	0.06014	0.05646	0.04652	0.04078	0.03568	0.02911	0.02062	13
14	0.06690	0.06260	0.05853	0.05467	0.05102	0.04130	0.03575	0.03087	0.02469	0.01689	14
15	0.06212	0.05783	0.05377	0.04994	0.04634	0.03683	0.03147	0.02682	0.02102	0.01388	15
20	0.04542	0.04116	0.03722	0.03358	0.03024	0.02185	0.01746	0.01388	0.00976	0.00536	20
25	0.03541	0.03122	0.02743	0.02401	0.02095	0.01368	0.01017	0.00750	0.00470	0.00212	25
30	0.02875	0.02465	0.02102	0.01783	0.01505	0.00883	0.00608	0.00414	0.00230	0.00085	30
35	0.02400	0.02000	0.01654	0.01358	0.01107	0.00580	0.00369	0.00232	0.00113	0.00034	35
40	0.02046	0.01656	0.01326	0.01052	0.00828	0.00386	0.00226	0.00130	0.00056	0.00014	40
45	0.01771	0.01391	0.01079	0.00826	0.00626	0.00259	0.00139	0.00074	0.00028	0.00005	45
50	0.01551	0.01182	0.00887	0.00655	0.00478	0.00174	0.00086	0.00042	0.00014	0.00002	50
60	0.01224	0.00877	0.00613	0.00420	0.00283	0.00080	0.00033	0.00013	0.00003		60
75	0.00902	0.00586	0.00367	0.00223	0.00132	0.00025	0.00008	0.00002			75
100	0.00587	0.00320	0.00165	0.00081	0.00038	0.00004	0.00001				100

TABLE 4. *Capital Recovery Factor; Given* P, *to Find* A

N	*i*=1%	*i*=2%	*i*=3%	*i*=4%	*i*=5%	*i*=8%	*i*=10%	*i*=12%	*i*=15%	*i*=20%	*N*
1	1.01000	1.02000	1.03000	1.04000	1.05000	1.08000	1.10000	1.12000	1.15000	1.20000	1
2	0.50751	0.51505	0.52261	0.53020	0.53780	0.56077	0.57619	0.59170	0.61512	0.65455	2
3	0.34002	0.34675	0.35353	0.36035	0.36721	0.38803	0.40211	0.41635	0.43798	0.47473	3
4	0.25628	0.26262	0.26903	0.27549	0.28201	0.30192	0.31547	0.32923	0.35027	0.38629	4
5	0.20604	0.21216	0.21835	0.22463	0.23097	0.25046	0.26380	0.27741	0.29832	0.33438	5
6	0.17255	0.17853	0.18460	0.19076	0.19702	0.21632	0.22961	0.24323	0.26424	0.30071	6
7	0.14863	0.15451	0.16051	0.16661	0.17282	0.19207	0.20541	0.21912	0.24036	0.27742	7
8	0.13069	0.13651	0.14246	0.14853	0.15472	0.17401	0.18744	0.20130	0.22285	0.26061	8
9	0.11674	0.12252	0.12843	0.13449	0.14069	0.16008	0.17364	0.18768	0.20957	0.24808	9
10	0.10558	0.11133	0.11723	0.12329	0.12950	0.14903	0.16275	0.17698	0.19925	0.23852	10
11	0.09645	0.10218	0.10808	0.11415	0.12039	0.14008	0.15396	0.16842	0.19107	0.23110	11
12	0.08885	0.09456	0.10046	0.10655	0.11283	0.13270	0.14676	0.16144	0.18448	0.22526	12
13	0.08241	0.08812	0.09403	0.10014	0.10646	0.12652	0.14078	0.15568	0.17911	0.22062	13
14	0.07690	0.08260	0.08853	0.09467	0.10102	0.12130	0.13575	0.15087	0.17469	0.21689	14
15	0.07212	0.07783	0.08377	0.08994	0.09634	0.11683	0.13147	0.14682	0.17102	0.21388	15
20	0.05542	0.06116	0.06722	0.07358	0.08024	0.10185	0.11746	0.13388	0.15976	0.20536	20
25	0.04541	0.05122	0.05743	0.06401	0.07095	0.09368	0.11017	0.12750	0.15470	0.20212	25
30	0.03875	0.04465	0.05102	0.05783	0.06505	0.08883	0.10608	0.12414	0.15230	0.20085	30
35	0.03400	0.04000	0.04654	0.05358	0.06107	0.08580	0.10369	0.12232	0.15113	0.20034	35
40	0.03046	0.03656	0.04326	0.05052	0.05828	0.08386	0.10226	0.12130	0.15056	0.20014	40
45	0.02771	0.03391	0.04079	0.04826	0.05626	0.08259	0.10139	0.12074	0.15028	0.20005	45
50	0.02551	0.03182	0.03887	0.04655	0.05478	0.08174	0.10086	0.12042	0.15014	0.20002	50
60	0.02224	0.02877	0.03613	0.04420	0.05283	0.08080	0.10033	0.12013	0.15003	0.20000	60
75	0.01902	0.02586	0.03367	0.04223	0.05132	0.08025	0.10008	0.12002	0.15000	0.20000	75
100	0.01587	0.02320	0.03165	0.04081	0.05038	0.08004	0.10001	0.12000	0.15000	0.20000	100

TABLE 5. *Compound Amount Factor (Uniform Series); Given* A, *to Find* F

N	*i*=1%	*i*=2%	*i*=3%	*i*=4%	*i*=5%	*i*=8%	*i*=10%	*i*=12%	*i*=15%	*i*=20%	N
1	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1
2	2.010	2.020	2.030	2.040	2.050	2.080	2.100	2.120	2.150	2.200	2
3	3.030	3.060	3.091	3.122	3.153	3.246	3.310	3.374	3.472	3.640	3
4	4.060	4.122	4.184	4.246	4.310	4.506	4.641	4.779	4.993	7.442	4
5	5.101	5.204	5.309	5.416	5.526	5.867	6.105	6.353	6.742	9.930	5
6	6.152	6.308	6.468	6.633	6.802	7.336	7.716	8.115	8.754	12.916	6
7	7.214	7.434	7.662	7.898	8.142	8.923	9.487	10.089	11.067	16.499	7
8	8.286	8.583	8.892	9.214	9.549	10.637	11.436	12.300	13.727	20.799	8
9	9.369	9.755	10.159	10.583	11.027	12.488	13.579	14.776	16.786	25.959	9
10	10.462	10.950	11.464	12.006	12.578	14.487	15.937	17.549	20.304	32.150	10
11	11.567	12.169	12.808	13.486	14.207	16.645	18.531	20.655	24.349	39.580	11
12	12.683	13.412	14.192	15.026	15.917	18.977	21.384	24.133	29.002	48.497	12
13	13.809	14.680	15.618	16.627	17.713	21.495	24.523	28.029	34.352	59.196	13
14	14.947	15.974	17.086	18.292	19.599	24.215	27.975	32.393	40.505	72.035	14
15	16.097	17.293	18.599	20.024	21.579	27.152	31.772	37.280	47.580	186.688	15
20	22.019	24.297	26.870	29.778	33.066	45.762	57.275	72.052	102.443	471.981	20
25	28.243	32.030	36.459	41.646	47.727	73.106	98.347	133.334	212.793	1181.881	25
30	34.785	40.568	47.575	56.085	66.439	113.283	164.494	241.332	434.744	2948.339	30
35	41.660	49.994	60.462	73.652	90.320	172.317	271.024	431.663	881.168	7343.9	35
40	48.886	60.402	75.401	95.026	120.800	259.057	442.593	767.088	1779.1	18281.3	40
45	56.481	71.893	92.720	121.029	159.700	386.506	718.905	1358.224	3585.1	45497.1	45
50	64.463	84.579	112.797	152.667	209.348	573.770	1163.909	2400.008	7217.7		50
60	81.670	114.052	163.053	237.991	353.584	1253.213	3034.816				60
75	110.913	170.792	272.631	448.631	756.654	4002.557	12708.954				75
100	170.481	312.232	607.288	1237.624	2610.025	27484.516	137796.123				100

TABLE 6. *Present Worth Factor (Uniform Series); Given* A, *to Find* P

N	*i*=1%	*i*=2%	*i*=3%	*i*=4%	*i*=5%	*i*=8%	*i*=10%	*i*=12%	*i*=15%	*i*=20%	*N*
1	0.990	0.980	0.971	0.962	0.952	0.926	0.909	0.893	0.870	0.833	1
2	1.970	1.942	1.913	1.886	1.859	1.783	1.736	1.690	1.626	1.528	2
3	2.941	2.884	2.829	2.775	2.723	2.577	2.487	2.402	2.283	2.106	3
4	3.902	3.808	3.717	3.630	3.546	3.312	3.170	3.037	2.855	2.589	4
5	4.853	4.713	4.580	4.452	4.329	3.993	3.791	3.605	3.352	2.991	5
6	5.795	5.601	5.417	5.242	5.076	4.623	4.355	4.111	3.784	3.326	6
7	6.738	6.472	6.230	6.002	5.786	5.206	4.868	4.564	4.160	3.605	7
8	7.652	7.325	7.020	6.733	6.463	5.747	5.335	4.968	4.487	3.837	8
9	8.566	8.162	7.786	7.435	7.108	6.247	5.759	5.328	4.772	4.031	9
10	9.471	8.983	8.530	8.111	7.722	6.710	6.145	5.650	5.019	4.192	10
11	10.368	9.787	9.253	8.760	8.306	7.139	6.495	5.938	5.234	4.327	11
12	11.255	10.575	9.954	9.385	8.863	7.536	6.814	6.194	5.421	4.439	12
13	12.134	11.348	10.635	9.986	9.394	7.904	7.103	6.424	5.583	4.533	13
14	13.004	12.106	11.296	10.563	9.899	8.244	7.367	6.628	5.724	4.611	14
15	13.865	12.849	11.938	11.118	10.380	8.559	7.606	6.811	5.847	4.675	15
20	18.046	16.351	14.877	13.590	12.462	9.818	8.514	7.469	6.259	4.870	20
25	22.023	19.523	17.413	15.622	14.094	10.675	9.077	7.843	6.464	4.948	25
30	25.808	22.396	19.600	17.292	15.372	11.258	9.427	8.055	6.566	4.979	30
35	29.409	24.999	21.487	18.665	16.374	11.655	9.644	8.176	6.617	4.992	35
40	32.835	27.355	23.115	19.793	17.159	11.925	9.779	8.244	6.642	4.997	40
45	36.095	29.490	24.519	20.720	17.774	12.108	9.863	8.283	6.654	4.999	45
50	39.196	31.424	25.730	21.482	18.256	12.233	9.915	8.305	6.661	4.999	50
60	44.955	34.761	27.676	22.623	18.929	12.377	9.967	8.324	6.665		60
75	52.587	38.677	29.702	23.680	19.485	12.461	9.992	8.333	6.666		75
100	63.029	43.098	31.599	24.505	19.848	12.494	9.999				100

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