DYNAMIC MODELS OF ILLEGAL BEHAVIOR

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November 1986

A REPORT TO THE NATIONAL INSTITUTE OF JUSTICE

84-IJ-CX-0028
UCSB# 08-291-84
4/1/84-6/30/86

Introduction

Existing models of criminal behavior are firmly rooted in the traditional neoclassical assumption that individuals maximize utility subject to a binding budget (and/or time) constraint, and that these decisions are made in a deterministic environment. Beginning with the work of Becker (1967), Ehrlich (1973), and Heineke (1978), economists have developed elaborate models to understand what motivates incentives for criminal behavior within this framework. For the most part, and regardless of the model's complexities, most of the results from this literature can be viewed as straightforward applications of the labor/leisure model of choice. Individuals allocate their time among two types of labor (criminal or legal activities) and leisure. Each of these activities has certain costs and benefits associated with it. For example, leisure is usually assumed to provide utility to an individual, but he must forego earnings (in either the legal or illegal sector) in order to purchase leisure time. Similarly, criminal behavior may lead to increased incomes, but the individual foregoes leisure time as well as increases the probability that he will be caught, fined, and sentenced for his indiscretions. For each of these activities, therefore, the individual will calculate the relevant trade-offs and choose the course of action that maximizes his utility.

Economic theory shows that changes in the costs and benefits associated with each of the activities generate both income and substitution effects on the individual's behavior. For instance, an increase in the legal wage rate will increase real income, thus increasing the individual's demand for leisure, and reducing his supply to legal activities. At the same time, this increase in the legal wage rate makes <u>not</u> being employed in the legal sector relatively more expensive, and thus will increase labor supply to the legal sector. Practically all theorems currently available that purport to explain how criminal behavior arises are applications of these important insights. Holding real income constant, an increase in the cost of committing illegal activities will reduce the incentives to supply labor to the illegal sector, while an increase in the incomes associated with illegal activities will increase the benefits of supplying labor to the illegal sector.

In a sense, these models borrow "too little" from neoclassical economic theory and thus lead to rather limited insights into criminal behavior. For the most part (even though this was not the way Becker originally thought of the problem), the models have tended to focus on the behavior of specific individuals. The notion of a marketplace where goods (both legal and illegal) are exchanged at competitive prices is implicit in the discussions, but it is rarely explicitly modeled. Individuals, for example, invest in legal activities such as education and on-the-job training. These investments become an asset to the individual. These assets, like those arising from investments from physical capital, can then be traded or rented in the marketplace, and create the incentives for further investments (or perhaps for reduced investments if the market for such asssets has turned sour). The point, however, is that it is difficult to explain the existence of legal human capital investments unless the environment where these investments are traded--i.e., the labor market--is described.

Similarly, individuals can acquire skills or goods that are considered illegal in a particular constitutional setting. These acquisitions are, in a sense, capital assets that the individual can now take into the marketplace where such assets are traded and sell or rent at a competitive price. Again the motivations for participating in the illegal sector will clearly depend on the characteristics of the market where trades in illegal goods and skills are conducted. The traditional models of criminal behavior in neoclassical theory have little to say about these exchanges or markets since they are concerned only with the behavior of a single individual, who (presumably) can sell all he owns at a constant price. This view, however, is rather incomplete. The same motivations and incentives that the "representative" person in the model has will clearly affect other individuals in the economy. Other persons, therefore, will also want to acquire legal training and education, as well as illegal goods and skills. The existence of markets for both legal and illegal goods (and skills) ensures that the behavior of different individuals is internally consistent, and solves the problem of what the capital value of the various assets are. The prices given by this solution will then motivate individuals to acquire legal and/or illegal skills and goods such that the supply of these activities equals the demand for them.

In this paper, we begin the modeling of this view of criminal behavior. Our approach is, of course, heavily influenced by the asset-pricing models of finance theory that have been developed in the last few years (see Lucas, 1978; LeRoy and LaCivita, 1981; and LeRoy 1982). These influential models have been quite useful in developing important insights in modern macroeconomics and finance theory, and have added to an increased understanding of the way that asset markets (such as the stock market) work. This approach, therefore, has important things to say about the way individuals allocate and invest their financial assets. We believe this approach also has important things to say about the way individuals allocate and invest their time and effort among competing alternatives.

The various models we will develop below have some essential characteristics that make them quite similar to the traditional models of asset-pricing in finance theory:

1. Investments are risky. The existence of uncertainty is a key feature in understanding the investment decisions made by individuals in allocating their financial assets. It is also a key aspect of the decision making process that individuals follow in deciding to which sector of the economy (i.e., the legal and/or illegal sectors) they should allocate their time and effort. Investments in both legal and illegal activities are likely to have uncertain returns. For example, investments in such legal activities as becoming a lawyer may be less profitable than expected if "too many" persons are also pursuing a similar career, or if structural changes occur in the political system. Similarly, investments in smuggling outlawed goods is also risky since the individual may be captured by border guards and sentenced to a few years in prison. In what follows we will assume that allocations of time and effort to illegal activities are inherently more risky than allocations of time and effort to legal activities. It would not be surprising to find, given this assumption, that risk-averse persons will avoid investments in illegal activities (unless they have big payoffs). The asset-pricing model, however, also implies that the high level of risk associated with criminal activity (due perhaps to the punishments inflicted by the legal system) deters not only investments in illegal assets but investments in legal assets as well. In other words, punishment for criminals affects not only criminals, but also lowers the average income of the non-criminal population.

2. Individuals can invest in either legal or illegal human capital. Asset accumulation means nothing but the foregoing of today's consumption in order to obtain higher consumption levels in the future. The notion of an investment process taking place is, therefore, central to an asset-pricing model. In our application of this approach to criminal activity, we introduce the

investment process by assuming that individuals can invest in human capital. Moreover, we assume (following Becker, 1965) that human capital can either be general (i.e., useful in both the legal and illegal sectors) or specific (useful only in one of the sectors, and particularly the illegal sector). These capital stocks are used to produce either legal or illegal commodities.

3. Markets exist where legal and illegal commodities can be sold or traded. All individuals in the economy are making similar investment decisions and allocating their time and effort to learning how to produce legal and/or illegal goods. These investments become assets because markets where these goods can be traded exist. In a sense, these markets become a place where risks can be diversified among many traders. The prices that are struck in these markets, of course, depend on the amount of relative uncertainty that investments in criminal activity carry. If, for example, law enforcers almost always catch the offender and impose a relatively high punishment, investments in illegal activities are quite risky, and hence illegal goods will command a relatively high price.

Although the previous literature on criminal behavior uses some of these features in their models, this analysis represents the first attempt to simultaneously incorporate all these aspects of the economy into the single coherent and consistent (as well as successful) framework provided by the asset-pricing model of finance theory. This is not the first time, however, that asset-pricing models have made their appearance in labor economics or in problems that guide the allocation of labor across sectors. Much of the implicit contracts literature (see the recent survey by Rosen, 1985) can be viewed as an application of some of the principles of asset-pricing models. In that literature, however, the main focus of analysis is the explanation of "sticky" wages and unsticky employment probabilities over the business cycle.

We believe that asset-pricing models provide a unique opportunity for investigating the allocation decision by individuals in institutional settings that have the key features listed above. The application of this model to the choice of criminal versus legal activities will be seen to expand our understanding of criminal behavior beyond the (almost tautological) results provided by the neoclassical labor/leisure model.

1. A PRODUCTION ECONOMY MODEL OF CRIME

I. Introduction

In this section we illustrate how the familiar asset pricing model of finance theory can yield useful insights into the problem of time allocation facing the representative individual. Suppose there are two sectors in the economy to which the individual can allocate time and effort and thus generate income: a legal sector and an illegal sector. The key feature that distinguishes the two sectors is the existence of risk in the illegal sector; the generation of illegal income carries the probability of being caught, fined, sentenced, etc. More generally illegal income is taxed, and the tax rate is a random variable.

One of the most important insights provided by the application of the asset-pricing model to this basic microeconomic problem is that the individual plays a double role in such an economy, that of a producer and that of a consumer. In this sense, therefore, the model has much in common with the Becker (1965) model of time allocation. Since the application of asset-pricing models and techniques to microeconomic allocation problems is likely to be unfamiliar to most readers, it is instructive to develop the simplest model at length.

II. The Production Decision

An individual plays two roles in the model: he generates income by producing and allocating time and effort to either of the two sectors, and he maximizes his utility by using the generated income in the purchase of goods produced in either of the two sectors. The individual is asssumed to have a fixed stock of human capital. At a given point in time, t, a fraction of this endowment, $k_{\ell t}$, is used to generate income in the legal sector, while the remaining fraction, k_{ct}, is used to generate income in the illegal (or "criminal" sector). The allocation of the fixed endowment into these two components is endogenous to the model.

Let L_{ti} be the level of output generated in the legal sector, and C_{ti} be the level of output generated by criminal activities, where the subscript i indexes the state of the world observed by the individual, and the various states of the world represent the riskiness of investments in criminal activity. We can then define the (inverse) production functions associated with these outputs by:

$$k_{\ell ti} = f(L_{ti}) \qquad (1)$$

$$k_{cti} = g(C_{ti}) \qquad (2)$$

The functions f and g are assumed to be linearly homogeneous.

Like firms, individuals use these production technologies to maximize the profits of allocating time and effort (as represented by the k's) to the two sectors of the economy. The profit functions for each of these types of activities are given by:

$$\pi_{\ell ti} = \max_L \, [L_{it} - w_{\ell ti} \, f(L_{it})] \qquad (3)$$

$$\pi_{cti} = \max_c \, [P_{it} \, C_{it} - w_{cti} \, (1 + \tau_i) \, g(C_{it})] \qquad (4)$$

where P_{it} is the relative price of criminal output, $w_{\ell ti}$ is the rental price of human capital used in the legal sector, and w_{cti} is the rental price of human capital used in the illegal sector. The price of output generated by activities in the legal sector is normalized to unity. Of course, in a competitive market the rental prices of the two types of human capital will be equated, and hence $w_{\ell ti} = w_{cti} = w_{ti}$. The parameter τ_i measures the tax rate that is levied on production in the illegal sector. This tax rate may measure the severity of the sentence or the size of the fine associated with the particular illegal activity. The tax rate, of course, is indexed by the subscript i since it is a random variable. The randomness is introduced by the fact that not all persons allocating time to criminal activity are caught. In principle, the tax rate can be viewed as an endogenous variable in a more general model that would take into account the maximization of government objectives, but in this simplest model we view the tax rate as determined by random shocks.

The specification of the profit functions in equations (3) and (4) captures one important aspect of the asset pricing model: a market for legal and illegal goods exist; this market generates the relative competitive price P_{it}; and at these prices the individual can sell all the goods he has produced in each of these two sectors. As will be seen below, the relative price of criminal goods will depend upon the nature of the uncertainty surrounding participation in the illegal sector.

Individuals maximizing the profits from allocating human capital to the criminal and legal sectors behave, in effect, like competitive firms. Hence the first order conditions associated with this maximization are:

$$P_{it} = (1 + \tau_i)\, w_{it}\, g'(C_{it}) \tag{5}$$

$$1 = w_{it}\, f'(L_{it}) \tag{6}$$

Equations (5) and (6) give the familiar result that the value of marginal product in a particular activity are equalized with the marginal costs associated with that activity. Taking the ratio of these two equations yields:

$$P_{it} = (1 + \tau_i)\ [g'/f'] \tag{7}$$

so that (for a given state of the world), the relative price of illegal goods will be determined by the weighted marginal rate of substitution between legal and illegal activities. Note that if individuals are equally efficient in producing legal and illegal goods, so that $g'(x) = f'(x)$, then the relative price of criminal goods will equal $1 + \tau_i > 1$. The existence of uncertainty in the production of illegal goods, therefore, leads to the market valuing criminal goods, C, at a higher rate than legal goods, L.

It is important to note that there exists a close relationship between this view of the human capital allocation model, and the developing literature on implicit contracts in labor economics (Rosen, 1985). In particular, the individual, as owner of his human capital, chooses to allocate his endowment between criminal and legal activities. Following Rosen, he contracts with himself as follows: "If θ takes on a value of θ_i, then I will supply $k_{\ell ti}$ units of human capital to legal activities, and the remainder to illegal activities." This type of individual behavior leads to "supply" functions:

$$k_{\ell ti} = \alpha(\tau_i) \tag{8}$$

$$k_{cti} = \beta(\tau_i) \tag{9}$$

By setting up this *ex ante* contract or agreement, the individual effectively eliminates the diversifiable risk induced by the stochastic nature of the tax structure. Such an implicit contract is based on complete information, thus ruling out any ex-post renegotiation of the terms. Moreover, due to the "specific" nature of the human capital allocated to the various activities, the individual finds it impossible to shift his accumulated human capital to sectors that, *ex post*, have become more profitable.

III. The Consumption Decision

The representative individual in this model is assumed to maximize the expected value of the discounted utility stream:

$$\text{Max } E \Sigma \beta^{t} U(C_{t}, L_{t}) \tag{10}$$

where β is the discount factor, and the expectation is taken over all possible states of the world. The utility function U in (10) is assumed to be strictly concave.

This specification of utility views criminal goods and legal goods as potentially different goods. More generally, the specification in (10) implies the existence of a preference structure over risky goods and nonrisky goods. This kind of breakdown can be easily understood if the criminal goods refer to such commodities as drugs, "hot" products, etc. Equation (10), however, also allows a more narrow interpretation of C and L. In particular, if criminal activities essentially define the way income was generated, such as gambling or tax evasion, and the legal activities refer to legal labor market earnings, then the utility function in (10) can be written as U(C+L) since the individual would then view the two types of incomes as perfect substitutes.

In order to derive the budget constraint associated with this maximization it is instructive to consider the possible source of incomes and expenditures that the individual can generate in the asset-pricing model. As in the implicit contracts literature, the market plays an important role in the model: it allows firms to "insure" individual risks. In the implicit contracts model of the labor market, this insurance takes the form of firms offering a fixed wage under all states of the world, but "bad" outcomes will generate layoffs for "unlucky" individuals. In the asset pricing model, the market plays an analogous role: Individuals can "trade" their "shares" of legal and illegal outputs at competitive prices. The prices at which these exchanges take place, of course, are endogenously determined within the model.

Given this interpretation of the process, consider the types of expenditures that the individual will undertake at time t. The maximization of lifetime utility generates demand functions for goods C and L (as opposed to the human capital supply functions generated by the production model). These demands lead to dollar expenditures of (L + PC) for the representative individual. In principle, however, the individual's allocation of effort and human capital modeled earlier need not lead to levels of legal and illegal outputs which are identical to those demanded within the current period. The existence of excess demands or excess supplies, as well as the agreements to supply certain levels of human capital to the production of goods in both sectors, will create a market wherein these contracts can be exchanged, and thus the risk facing any given individual can be diversified.

In particular, let $Z_{\ell ti}$ be the share of total legal output available in period t which the individual wishes to "reserve" for future consumption, and Z_{cti} be the share of total illegal output also reserved by the individual for future consumption. By definition, the Z's add up to unity across persons in the market. The "futures" market will lead to competitive prices of $\phi_{\ell ti}$ and ϕ_{cti}, repectively, for shares of legal and illegal goods. These prices, of course, are nothing but the Arrow-Debreu price of contingent claims.

It is insightful to again interpret the structure of the model in terms of the implicit contracts literature. The consumption model outlined in this section generates demand fucntions for legal and illegal goods, while the production function outlined earlier fixes the number of such goods available in the market place. The representative individual enters a contract to supply a certain number of both types of goods. The individual can either sell this contract in a competitive market, or he could impute a value to it. The equilibrium conditions of the model are derived by imposing the market clearing (or consistency) restrictions that all output is consumed. The value of the contingent claim ϕ, therefore is nothing but the price of this contract, and will, in general, reflect two things: (1) the production technology which determines the optimal allocation of human capital between the legal and illegal sectors (for a given state of the world i); and (2) the characteristics of the individual's utility function, i.e., his risk aversion and the rate of time preference.

At each point in time then individuals hold these claims to output which were acquired in the previous period and they receive income from their endowment of labor. For convenience we normalize the endowment at unity. This provides an income to the individual equal to w_{it}. In period t-1 the individual purchased $Z_{\ell j,t-1}$ and $Z_{c,t-1}$; share of legal and illegal goods, respectively. These shares (purchased in t-1) generate two types of incomes in period t: a "dividend" on the contingent claims, $\pi_{\ell it}$ and π_{cti}; and an opportunity cost of $\phi_{\ell ti}$ and ϕ_{cti}, since these shares could be resold in the market at these prices. Hence the budget constraint facing the representative individual is given by:

$$W_{it} + Z_{cj,t-1}(\pi_{ctj} + \phi_{cti}) + Z_{\ell,t-1,j}(\pi_{\ell,tj} + \phi_{\ell j})$$

$$= L_{ti} + PC_{ti} + \phi_{cti}Z_{cti} + \phi_{\ell ti}Z_{\ell ti}$$

IV. Solution

The solution of this problem is obtained by dynamic programming. In the steady state, the various prices in the model will depend on the state of nature and not on the date, hence we can eliminate the time subscript in what follows.

The general solution to this problem is not instructive unless further structure is imposed on the dynamic characteristics of the uncertainty. The simplest possible structure that can be imposed is one where the tax rate, τ, can be characterized by a stationary two-state Markov process with the tax rates taking on a value of 0 or θ. The conditional probabilities associated with the dynamic behavior of the tax rate over time are $P(0|i)$ and $P(\theta|i)$, for $i = 0$ or θ. These conditional probabilities denote the probability that the next state will be 0 or θ, given that the current state is 0 or θ. This simple characterization of the uncertainty incorporates the intuitive notion that in the production and consumption of illegal goods one may not be caught (hence the observed tax rate is zero), or if one is caught the penalty will be given by tax rate θ. Moreover, the conditional probabilities allow the individual's luck to be correlated over time.

Under this simple stochastic structure, the first-order conditions for the maximization of utility are given by:

$$(\partial U/\partial L)_i \phi_{\ell i} = \beta[P(0|i)(\partial U/\partial L)_0(\Pi_{\ell 0} + \phi_{\ell 0}) + P(\theta|i)(\partial U/\partial L)_\theta(\Pi_{\ell\theta} + \phi_{\ell\theta})] \quad i=0,\theta \tag{11}$$

$$(\partial U/\partial C)_i \phi_{\ell i} = \beta[P(0|i)(\partial U/\partial C)_0(\Pi_{c0} + \phi_{c0}) + P(\theta|i)(\partial U/\partial C)_\theta(\Pi_{c\theta} + \phi_{c\theta})] \qquad i=0,\theta \tag{12}$$

$$\frac{U_{C,i}}{U_{L,i}} = P_{c,i} \qquad i=0,\theta \tag{13}$$

These first-order conditions have the traditional interpretation that the expected marginal gain (in utility) is equal to the expected marginal cost of the foregone consumption.

By imposing the Markovian structure on the probabilities of the various states, it is possible to use the first-order conditions (11)-(13) to solve for the equilibrium prices of the contingent claims. These equilibrium prices are given by:

$$\phi_{c0} = \frac{\beta\Pi_{c0}\{P(0|0)-\beta[P(0|0)+P(\theta|\theta)-1]\}+\beta\Pi_{c\theta}\lambda[1-P(0|0)]}{(1-\beta)\{1+\beta-\beta[P(0|0)+P(\theta|\theta)]\}} \tag{14}$$

$$\phi_{c\theta} = \frac{\beta\Pi_{c0}\lambda^{-1}[1-P(\theta|\theta)]+\beta\Pi_{c\theta}\{P(\theta|\theta)-\beta[P(0|0)+P(\theta|\theta)-1]\}}{(1-\beta)\{1+\beta-\beta[P(0|0)+P(\theta|\theta)]\}} \tag{15}$$

where $\lambda = U_{c,\theta}/U_{c,0}$. It is seen that these equilibrium prices depend on the stochastic properties of transition across states and on the characteristics of the representative individual's utility function. Similar equations hold for $\phi_{\ell 0}$ and $\phi_{\ell\theta}$. Given the structure of the model, the supplies of criminal (and legal) activities can be determined solely from the behavior of these price functions.

V. Implications

The model developed above leads to a number of predictions about the relationships between human capital investment, risk aversion, punishment probabilities and endowments (initial wealth) in a world of uncertain returns. For the most part these correlations simply accord with common sense but it is useful to review them here.

Change in Risk Aversion

The equilibrium price equations (14) and (15) show that ratio of the shadow prices of human capital ($\phi_{c0}/\phi_{c\theta}$) is positively related to $\lambda = U_{c,\phi}/U_{c,0}$. If we assume a utility function of the constant relatve risk aversion class,

$$U(c) = \frac{C^{1-\gamma}-1}{1-\gamma} \quad , \cdot\gamma>1$$

where γ is a measure of risk aversion. It is clear that the more risk averse individuals are the less they value illegal activities when the tax state is high. By construction they will invest more of the human capital in legal endeavors.

Change in Serial Correlation of the Tax State

The tax state being high (θ) or low (0) reflects the extent to which illegal activities are being punished. The degree of serial correlation in the state simply reflects the probability of the current state persisting and thus helps individuals predict what the probability of the states will be next period, given what they are currently. If the states are independent $P(0|0) = P(\theta|\theta) = .5$, and the state has no influence on the decision to allocate human capital to illegal activities. If there is positive serial correlation, $P(0|0) + P(\theta|\theta) > 1$, the individual agents will be more likely to allocate capital to illegal activities when the current tax state is low and less likely to allocate it when the current tax state is high. Conversely, when $P(0|0) + P(\theta|\theta) < 1$, a currently low tax state signals that the next tax is more likely to be high and individuals will be less likely to allocate capital to illegal activities.

Effect of a Change in Endowment

Note that, the level of endowment is given by the aggregate human capital, $\bar{K}$ available to the agents. We are interested in knowing the effect of a change in $\bar{K}$ on the endogenous variables. It is difficult to provide general results, but it is possible to get unambiguous results with a specific parameterization.

Assume that the production functions are Cobb-Douglas and the utility function is logarithmic. So we have:

$$f(L_i) = L_i^{\frac{1}{\delta}};\ 0 < \delta < 1;\quad i=0,\theta, \tag{16}$$

$$g(C_i) = C_i^{\frac{1}{\alpha}};\ 0 < \alpha < 1;\quad i=0,\theta. \tag{17}$$

$$U(C_i,L_i) = \log C_i + \text{Log } L_i\ . \tag{18}$$

With these functional forms the equilibrium capital allocation is given by:

$$f(L_i^*) = \frac{(1+T_i)\delta^{\frac{1}{1-\delta}} \cdot \bar{K}}{\alpha \cdot \delta^{\frac{\delta}{1-\delta}} + (1+T_i)\delta^{\frac{1}{1-\delta}}}\ ;\ \text{for } i=0,\theta \tag{19}$$

$$g(C_i^*) = \frac{\alpha\ \delta^{\frac{\delta}{1-\delta}} \cdot \bar{K}}{\alpha \cdot \delta^{\frac{\delta}{1-\delta}} + (1+T_i)\delta^{\frac{1}{1-\delta}}} \text{ ; for } i=0,\theta \tag{20}$$

Comments:

(a) This immediately shows that both $f(L_i^*)$ and $g(L_i^*)$ are positive implying that there is no corner solution. In fact, the corner solution is ruled out if we assume that $U'(0,\cdot)\to\infty$ and $U'(\infty,\cdot)\to 0$, $U'(\cdot,0)\to\infty$ and $U'(\cdot,\infty)=0$. This implies that the indifference curves relating C_i and L_i can never touch the axis. Hence, it is impossible to have a situation where $C_i = 0$ meaning $g(C_i) = 0$. The logarithmic utility function satisfies these restrictions.

(b) Note that $f(L_i^*)$ is a monotonically increasing function of θ and $g(C_i^*)$ is monotonically decreasing in θ. In other words, when the tax rate changes from 0 to θ, less human capital is allocated to criminal activity and more is allocated to legal activity. This is not surprising. It follows from the concavity of the p.d.f. between L_i and C_i.

(c) Finally, when $\bar{K}$ increases capital allocation is higher in both activities. The relative increase in the allocation of capital in any activity depends on the production parameters α,δ and the tax parameter, θ.

The steady state price function, ϕ_0^c and ϕ_τ^c are given by equations (14) and (15). For logarithmic utility functions the parameter λ has the form:

$$\lambda^* = \frac{U_{C,\theta}}{U_{L,0}} = \frac{C_0^*}{C_\tau^*} = \frac{\alpha\delta^{\frac{\delta}{1-\delta}} + (1+\tau)\delta^{\frac{1}{1-\delta}}}{\alpha\delta^{\frac{\delta}{1-\delta}} + \delta^{\frac{1}{1-\delta}}}, \tag{21}$$

$$\pi_{c0}^* = \frac{\delta^{\frac{\delta}{1-\delta}} - \alpha\delta^{\frac{\delta}{1-\delta}}}{[\alpha,\delta^{\frac{\delta}{1-\delta}} + \delta^{\frac{1}{1-\delta}}]^{\delta}} \cdot \bar{K}^{\delta}. \tag{22}$$

$$\pi_{\ell\theta}{}^{*} = \frac{\delta^{\frac{\delta}{1-\delta}} - \alpha\delta^{\frac{\delta}{1-\delta}}}{\left[\frac{\alpha\delta^{\frac{\delta}{1-\delta}}}{(1+\theta)} + \delta^{\frac{1}{1-\delta}}\right]^{\delta}} \cdot \bar{K}^{\delta}. \tag{23}$$

Comments:

It can be immediately verified that both $\frac{\partial\phi_{C0}}{\partial\bar{K}}$, $\frac{\partial\phi_{C\theta}}{\partial\bar{K}}$ are positive. This is due to the fact that $\frac{\partial\Pi_i{}^{*c}}{\partial\bar{K}} > 0$ for $i=0,\theta$ and the utility function belongs to the CRRA charts, implies that λ is independent of the wealth effect. Hence the sign of $\frac{\partial\Pi_i{}^{*c}}{\partial\bar{K}}$ directly determines the sign of $\frac{\partial\phi_i{}^{*c}}{\partial\bar{K}}$ for $i=0,\theta$.

Hence our conclusion is that the contingent claim prices attached to criminal activity will increase in both states when the endowment in the economy is higher.

2. An Economy With Investment in Human Capital

I. Introduction

In the previous chapter we analysed the allocation of human capital between illegal and legal activities in an economy where the total stock of human capital is fixed. Obviously an important element of individual decision making is overlooked because an essential part of every decision plan is to decide how much capital to accumulate. In the standard static economic analysis this is typically characterised as a decision to acquire more education or on the job training at the expense of current consumption. Here, we take an approach similar to that used in the previous section: we view the problem of human capital allocation and aquisition in an abstract setting where the individual is both consumer and producer. In this framework we examine the decision to invest additional resources in the accumulation of human capital, the total of which can be allocated to both legal and illegal activities.

As in the previous chapter we assume that there are two production processes, one for legal goods and services, the other for illegal goods and services. Again, the fact that illegal activities are not sanctioned by society is captured by the assumption that the activity is taxed at some rate τ which is stochastically shifting over time. We also assume that legal activities are subject to a random return. This is meant to capture the feature that the returns to legal activity may vary over time because the output is sensitive to the business cycle. We introduce this type of uncertainty through a random shock to the production technology of the legal sector.

Individuals in this economy face the following sequence of decisions: at the beginning of each period the tax or punishment state and the technological or employment state is revealed. This determines the climate for legal and illegal activity and the individual decides how to allocate his existing human capital K_t to the two activities. Having decided on the allocation of existing capital, he must then make a consumption-saving decision with respect to the total output. The latter decision determines the level of his capital stock in the future. As in the previous analysis, we have adopted a very simple structure that captures the dynamic elements of a much more complicated reality. Our goal is to analyze the allocation of human capital to both of these activities and to study the decision to invest. As a by-product we examine the behavior of the shadow price of the human capital allocated to legal and illegal activities.

II. Production

With perfect knowledge about the stock of human capital and the current realization of the state variables, the individual as producer maximizes profits from the production of the two outputs. We assume a constant returns to scale production technology and we assume further that there is a specific input to each production technology. The maximization problem faced by individual producers is:

$$\pi_t^{\ell} = \max_{K_{1t}} [\varepsilon_t F_1(K_t^{\ell}, S_t^{\ell}) - r_{\ell}(K_t, \tau_t, \varepsilon_t) \cdot K_t^{\ell}]$$

$$\pi_t^{c} = \max_{K_{1t}} [(1-\tau_t) F_2(K_t^{c}, S_t^{c}) - r_{c}(K_t, \tau_t, \varepsilon_t) \cdot K_t^{c}]$$

where π_t^{ℓ} = profit from the legal activity (ℓ)

π_t^{c} = profit from the illegal activity (c)

K_{1t} = capital allocated to legal sector

K_{2t} = capital allocated to illegal sector

$F_1(\cdot,\cdot)$ = constant returns to scale production technology for the legal sector

$F_2(\cdot,\cdot)$ = constant returns to scale production technology for the illegal sector

S_t^{ℓ} = specific input used in the production of legal activity

S_t^{c} = specific input used in the production of illegal activity

$r_{\ell}(\cdot)$ = shadow rental rate for the human capital in the ℓ sector

$r_c(\cdot)$ = shadow rental rate for the human capital in the c sector

Here ε_t and τ_t are random variables with $E(\varepsilon_t) = 1$. The shadow rental rates for human capital are functions of the total capital stock K_t and the random variables. The only non-trivial decision problem is the allocation of K_t between the two sectors conditional on a realization of ε and τ.

III. Consumers Decisions

The representative consumer in this economy is assumed to choose a path of consumption of the two goods to maximize his expected discounted lifetime utility:

$$\text{(3.1)} \qquad \text{Max } E_0 \sum_{t=0}^{\infty} \beta^t [U(C_t) + W(L_t)]$$

where U and W are strictly concave and β is the discount factor. The individuals face a budget constraint:

$$\text{(3.2)} \qquad \text{s.t.} \quad S_{t-1}^{\ell}(\pi_t^{\ell} + \phi_t^{\ell}) + S_{t-1}^{c}(\pi_t^{c} + \phi_t^{c}) + r_t^{\ell}K_t^{\ell} + r_t^{c}K_t^{c}$$

$$= L_t + P_tC_t + \phi_t^{L}S_t^{L} + \phi_t^{C}S_t^{C}$$

where ϕ_t^{ℓ} = implicit price of the specific input used in the legal activity

ϕ_t^{c} = implicit price of the specific input used in the illegal activity.

The left hand side of (3.2) represents an individual's wealth. The first two terms are simply the returns to individual holdings of the specific inputs. In this framework we may view that as specific human capital. "Production profits" are simply the returns to the specific factors. The remaining two terms on the left hand side are the returns to the rental of shiftable human capital in the two activities. The right hand side represents the way the wealth is allocated between current consumption ($L_t + C_t$) and holdings of the specific inputs to be used in the subsequent period. Although he is free to choose any level of S_t^{ℓ} and S_t^{c} in practice we assume that these inputs are available in fixed supply $\bar{S}_t^{\ell}$, $\bar{S}_t^{c}$ for all t and that $S_t^{\ell} = S_t^{c} = 1 \; \forall t$. This implies then that the implicit prices ϕ_t^{ℓ} and ϕ_t^{c} will move around to reflect changes in the desired allocation of specific inputs.

An equilibrium of this model economy will be characterized by the following consistency requirements.

(3.3) $$S_t^{c} = \bar{S}_t^{c} = S_t^{\ell} = \bar{S}_t^{\ell} = 1$$

(3.4) $$K_{1t} + K_{2t} = K_t$$

(3.5) $$r_{\ell}(K_t, \varepsilon_t, T_t) = \varepsilon_t \cdot f_1'(K_t^{\ell})$$

(3.6) $$r_c(K_t, \varepsilon_t, T_t) = (1-\tau_t) f_2'(K_t^{c})$$

where

$$f_1(K_t^{\ell}) = F\left(\frac{K_t^{\ell}}{S_t^{\ell}} , 1\right)$$

$$f_2(K_t^{c}) = F\left(\frac{K_t^{\ell}}{S_t^{\ell}} , 1\right)$$

by virtue of the constant returns technology and condition (3.3).

IV. A Growth Model

The model just described has many of the same features as that considered in the previous chapter, but it also introduces a technology shock ε_t and growth of the capital stock K_t. These features make it impossible to solve the model directly. Instead we can study the equilibrium of this model economy indirectly by constructing an optimal growth model which reproduces the equilibrium law of notion of the human capital, K_t. Such a growth model does not involve the shadow prices, r^{ℓ}, r^c, ϕ^{ℓ}, ϕ^c and hence it is easy to solve in a smaller state space.

Consider a simple economy where a single consumer decides about contingency plans for $\{C_t\}$, $\{L_t\}$, $\{K_t\}$ treating $\{\tau_t\}$, $\{\varepsilon_t\}$ as parametric. The consumer, therefore, solves the following problem:

(4.1) $$\max E_0 \sum_{t=0}^{\infty} \beta^t \cdot [U(C_t) + W(L_t)]$$

subject to

(4.2) $$C_t + L_t + K_{t+1} = (I-\tau_t)\ f_2(K_t^c) + \varepsilon_t f_1(K_t^{\ell})$$

(4.3) $$K_{1t} + K_{2t} = K_t$$

(4.4) $$K_t \geq 0\ ,\ K_{1t} \geq 0\ ,\ K_{2t} \geq 0$$

(4.5) $$\tau_t = \rho\tau_{t-1} + u_t \qquad \text{where} \qquad 0 < u_t < b$$
$$0 < b + \rho < 1$$

(4.6) ε_t is i.i.d.

The decision problem captures above combine the production and consumption decisions of the household. One could think of this growth model as the problem of a central planner while the model economy described earlier decision making as decentralized.

Rather than go through all of the algebra associated with this model economy in the abstract, we can illustrate the equilibrium of the model by considering some specific parametric forms for the preference and technology. We assume U and W are logarithmic

$$U(C_t) = \log C_t,$$

(4.8)

$$W(L_t) = \log L_t,$$

while the production technology can be written as

$$f_1(K_t^{\ell}) = K_t^{\ell}\, \alpha,$$

(4.9)

$$f_2(K_t^{c}) = K_t^{c}\, \alpha, \qquad 0 < \alpha < 1.$$

With this particular specification, we obtain the following decision rules:

(4.10a) $$K{*}_t^{\ell} = \bar{\eta}_1 K_t^{*}$$

(4.10b) $$K{*}_t^{c} = \bar{\eta}_2 K_t^{*}$$

where $0 < \bar{\eta}_i < 1$ and $\bar{\eta}_1 + \bar{\eta}_2 = 1$. The terms $\bar{\eta}_1$ and $\bar{\eta}_2$ are fractions of the equilibrium capital stock allocated to legal and illegal activities. Equations (4.10a) and (4.10b) imply a linear allocation rule for the stock of human capital.

The evaluation of the optimal stock of human capital is given by:

$$(4.11) \qquad K^*_t = \alpha\beta[\varepsilon_t \bar{\eta}_1^{\alpha} + (1-T_t)\ \bar{\eta}_2^{\alpha}]\ K^{*\alpha}_{t-1}.$$

Consumption and the returns to specific inputs evolve according to:

$$(4.12) \qquad L^*_t = C^*_t = \tfrac{1}{2} \cdot [1-\alpha\beta][\varepsilon_t \bar{\eta}_1^{\alpha} + (1-\tau_t)\ \bar{\eta}_2^{\alpha}]K_t^{\alpha}\ .$$

$$(4.13) \qquad \pi^{\ell}_t = (1-\alpha)\ K_t^{L^{\alpha}} \cdot \varepsilon_t,$$

$$(4.14) \qquad \pi^{c}_t = (1-\alpha)\ K_t^{c^{\alpha}} \cdot (1-\tau_t)$$

The optimal values of $\bar{\eta}_i$ can be determined by solving the following problem:

$$\max_{0 < \eta_i < 1} \iint [\log C^*_t + \log L^*_t]\ dF(\varepsilon_t)dG(\tau_t|\tau_{t-1})$$

$$\text{s.t.} \qquad \bar{\eta}_1 + \bar{\eta}_2 = 1.$$

where $F(\cdot)$ and $G(\cdot)$ are the probability measures for ε_t and τ_t respectively. Note that, due to serial correlation in τ_t, $G(\cdot)$ depends on the conditioning set, τ_{t-1}.

The solution to this problem yields optimal values of $\bar{\eta}_i$ which depend on the key parameters in a highly non-linear fashion. It is impossible to obtain an analytical expression for $\bar{\eta}_i$. It is possible to perform based simulation exercise drawings of ε_t and u_t as from an assumed distributions. We conducted such a simulation using a program written in the GAUSS language. It assumed a rectangular distribution for ε and u_t. That simulation plus an analysis of the equilibrium paths (4.11), (4.12) leads to the following conclusions:

1. Investment in general human capital decreases with increased uncertainty about the earnings from either legal or illegal activity. Increased uncertainty about either earnings stream leads to more current consumption and less investment.

2. The level of the tax on illegal activity affects both the stock of human capital and consumption of both goods inversely. That is, higher tax implies lower consumption.

3. The allocation of shiftable human capital ot legal activities is greater ($\bar{\eta}_1$ is greater) if $\frac{VAR(\tau)}{VAR(\varepsilon)}$ is higher. This has two interesting implications: If the uncertainty of the income stream from illegal activities is increased then, not surprisingly, more capital will be allocated to legal activities. Perhaps more importantly, if the relative uncertainty of the returns to legal activity increase, as they may do naturally over the declining phase of the business cycle, then the share of capital devoted to illegal activities will increase.

4. The implicit price of the specific input allocated to illegal activities, ϕ_t^c , is higher (lower) if the relative uncertainty of legal activities increases (decreases). This is simply a corollary of the preceding result but it reflects the fact that the specific input is valued more highly in this case.

These results accord well within intuition and observation. In the next section we study a model that attempts to further highlight the decisions to invest in human capital.

3. A Model With Overlapping Generations

I. Introduction

In the previous two chapters we analyzed models in which individuals make decisions about the allocation and accumulation of human capital for legal and illegal activities so as to maximize their expected lifetime utility of consumption. A key feature of those models is that individuals are assumed to be identical. Moreover, they are assumed to maximize their utility and form their expectations over an infinite horizon. These assumptions are employed because they make the analysis extremely tractable. The assumption that all individuals are identical implies that no trades actually take place but the prices supporting the equilibrium can easily be analyzed.

We now want to expand on the previous analysis by allowing for some heterogeneity of individuals. The easiest framework in which to do this is in a model with overlapping generations of individuals. This framework has been successfully used in many other areas of economics to introduce heterogeneity of individuals in the simplest possible way. In addition to introducing heterogeneity this model also has the feature that individuals have a finite horizon over which they maximize their utility. For the most part the conclusions of this analysis will be similar to those we derived from the previous models. There is little difference in the qualitative conclusions that come from introducing the finite time horizon or this simple form of heterogeneity. The model does however, produce a vivid picture of how individuals faced with risky earnings streams allocate time for the investment in various forms of human capital. We also illustrate how these decisions vary with risk aversion and with the characteristics of the uncertain reward stream.

II. The Model.

In this model we abstract from the characteristics of production that were made explicit in the previous sections. They could be introduced but would not add much to the analysis of this chapter where we want to focus on different issues. The economy we envision consists of a sequence of overlapping generations of individuals. Individuals live for two periods and in each period a new cohort is born so that at each point in time there is a generation of young and a generation of old. The young are born with a fixed endowment of time denoted $\bar{T}$ that they can allocate either to current market activity or to investment in training. Current market activity means participation in the labor market at a fixed competitively determined wage rate ($\bar{W}$) which we can think of as the wage for entry level youth employment. The alternative to current employment and consumption is to invest in human capital. We assume that the consumption opportunities of the old generation are determined entirely by the investment decisions they made when they were young. There are two possible investments: 1) human capital that will pay off with a certain return and, 2) human capital that will have a random, state dependent, return. Investing in the former can be thought of as buying an implicit contract to engage in legal activity with known income, while investing in the latter represents purchasing as implicit contract to engage in illegal activity with a random income stream.

At each point in time a generation is born and receives its initial endowment of time. They observe the current plight of the older generation, that is, the return they receive on their risky (illegal) human capital. Based on that observation, the young decide on the allocation of time between current market activity and the acquisition of the two forms of human capital that will determine their income when they are old. Our objective is to analyse the desired allocations of the initial endowment to these two activities. We denote these allocations T_c and T_ℓ respectively. In equilibrium the desired allocations and the actual allocations will be equal and we can generate shadow prices that value the scarce time resource. These shadow prices, which we denote P_c and P_ℓ, are essentially the value of implicit contracts to engage in criminal and legal activities in the future.

The uncertainty that individuals face is uncertainty over the return to the risky implicit contract. The contract is risky because of punishment probabilities and uncertain returns. We assume that the return to the risky implicit contract is determined by the realization of a stationary Markov process and that the parameters of that process are known to the young generation. If we assume for simplicity that the process has two states then we can denote the returns to the contract as R_c^0 and R_c^1 in the low and high states respectively. Individuals observe the current state when they are born (i.e. they observe the current payoff to illegal contracts) and must base their decisions on their prediction of the state that will prevail in the future given their knowledge of the transition probabilities π_{10}, π_{01}. We assume that the returns to implicit contract for legal activities is R_ℓ with certainty and that $R_c^1 > R_\ell > R_c^0$. That is, in the best state crime pays better but in the bad state it does not.

The above description is summarized in the following two period optimization problem that consumers face:

(3.1) $$\text{Max } U(Z_y^t) + E_t\, V(C_0^{t+1})$$

subject to

(3.2) $$Z_y^t = \bar{W}\, T_m^t$$

(3.3) $$T_m^t = \bar{T} - T_c^t - T_\ell^t$$

(3.4) $$Z_0^{t+1} = \tilde{R}_c \cdot T_c^t + R_\ell \cdot T_\ell^t$$

where:

Z_y^t - consumption of the young generation in time t.

Z_0^{t+1} - consumption of the old generation in t+1.

$\bar{W}$ - exogenously determined market wage.

$\bar{T}$ - total endowment of time of the young.

T_c^t - desired investment of time in acquiring human capital for illegal activities.

T_ℓ^t - desired investment of time in acquiring human capital for legal activities.

R_ℓ - return to legal human capital.

$\tilde{R}_c$ - state contingent return to illegal human capital. $\tilde{R}_c = \{R_c^1, R_c^0\}$.

Equations (3.1) - (3.9) will provide a solution for the allocation of time to legal and illegal activities in equilibrium. The solution is best characterized by considering the shadow prices of time invested in acquiring the legal and illegal human capital. With suitable restrictions on the utility function $U(Z_y)$ and $V(Z_0)$ that guarantee an interior solution the first order conditions of the optimization problem can be written as:

(3.5) $$P_c^t \cdot U'(Z_y^t) = E_t \cdot (V'(Z_0^{t+1}) \cdot \tilde{R}_c)$$

(3.6) $$P_\ell^t \cdot U'(Z_y^t) = E_t(V'(Z_0^{t+1}) \cdot R_\ell).$$

Equations (3.5) and (3.6) simply show that the shadow prices of time allocated to the acquisition of the two types of human capital adjust to equate the expected marginal utility of the returns from the consumption streams made possible by the two investments.

Because we have assumed the underlying source of uncertainty is determined by a stationary stochastic process we can characterize the equilibrium in terms of the states without reference to the dates. Hence we will drop the time subscripts. If we impose the equilibrium condition $T_c^t = \bar{T}_c$ and $T_\ell^t = \bar{T}_\ell$ we can rewrite the constraint (3.3) as

$$(3.7) \qquad T_m^i = \bar{T} - P_c^i \bar{T}_c - P_\ell^i \bar{T}_\ell,$$

so it is explicit that the shadow prices adjust to the state. This allows us to derive the stationary state contingent shadow price functions:

$$(3.8) \qquad P_c^i \cdot U'(Z_y^i) = \sum_{j \varepsilon \{0,1\}} \pi_{ij} R_c^j \cdot V'(R_c^j \bar{T}_c + R_\ell \bar{T}_\ell) \qquad i = 0,1$$

$$(3.9) \qquad P_\ell^j \cdot U(Z_y^i) = \sum_{j \varepsilon \{0,1\}} \pi_{ij} \quad V'(R_c^j \bar{T}_c + R_\ell \bar{T}_\ell) \qquad i = 0,1$$

Thus, there are four nonlinear price equations. With general functional forms for U( ) and V( ) it is difficult to characterize the solution. In the following section we impose some specific functional forms and discuss some of the results of our simulations.

III. Numerical Simulations

Our ultimate goal in this analysis is to understand how uncertainty about the returns from illegal activity influences the decision of the young to allocate time between current market activity and the acquisition of skills for both legal and illegal activity. Moreover, we are interested in how this allocation varies with the risk aversion of individuals and with the nature of the uncertainty. The results we obtain should be regarded simply as stylized outcomes that reflect the features and perhaps the limitations of this artificial economy.

We assume that the utility functions have the constant relative risk aversion forms

$$U(Z_t) = \frac{Z_t^{1-\gamma_1}}{1-\gamma_1} - 1 \qquad V(Z_t) = \frac{Z_t^{1-\gamma_2}}{1-\gamma_2} - 1$$

where $1 \leq \gamma_1, \gamma_2 < \infty$. The parameters γ_1, γ_2 represent the elasticity of substitution in consumption and increasing values of γ_1, γ_2 are associated with increasing risk aversion. In order to simulate and solve equations (3.8) and (3.9) we have to specify the transition probabilities governing the states and the relative returns. For the former we assume $\pi_{00} = \pi_{11} = .75$ which is the probability of the current state persisting when the current young generation is old. We assume further that $\bar{W} = 1.0$ $R_c^0 = 0$ $R_c^1 = 1.5$ and $R_\ell = 1.0$. Thus, a high state realization implies that the returns to illegal human capital is 50% higher than the returns to legal human capital while in the low state there is no return. This difference in returns could reflect either variation in the returns to legal human capital because of the business cycle or variations of the returns to illegal human capital because of law enforcement or any combination of factors. The important feature is that the relative returns vary.

The results of one simulation are summarized in Table 1 and there are several striking features which we summarize.

1. When the young generation observe low current returns to illegal human capital they devote 6% of their time to acquiring those skills under risk neutrality.

2. When the young observe a high current return to illegal capital the percentage of time invested in acquiring those skills increases dramatically to 15%.

3. Most of the variation in time allocated to acquiring illegal skills is a reallocation from current entry level employment rather than a substitution away from the acquisition of legal human capital.

4. The acquisition of illegal skills is <u>very</u> sensitive to risk aversion. The percentage of time devoted to that declines from 15% to 5% in the high state when the relative risk aversion increases from 1.0 to 1.9.

5. As the degree of risk aversion increases individuals substitute away from acquisition of illegal skills for the future and toward the acquisition of legal skills for the future with current employment relatively unaffected.

Table 2 presents the results of some numerical simulations that examine the effect of the structure of the uncertainty on the allocation of time. In these simulations we fixed the degree of risk aversion at $\gamma_1 = \gamma_2 = 1.5$ and examined the effect of increasing the probability of the current state continuing. The two cases of most interest here are $\pi_{00} = \pi_{11} = .5$ which implies that the states are equally likely and independent and $\pi_{00} = \pi_{11} = .99$ which implies that the current state is virtually certain to be realized when the current young generation is old. In the former case the young devote 6% of their time to the acquisition of illegal skills regardless of the current state. In the latter

TABLE 1

Time Allocation Under Increasing Risk Aversion

Risk Aversion	Fraction of Time Devoted to Acquiring Illegal Skills		Fraction of Time Devoted to Acquiring Legal Skills		Fraction of Time Devoted to Entry Level Employment	
	(L)	(H)	(L)	(H)	(L)	(H)
$\gamma_1 = \gamma_2 = 1.0$	.0647	.1525	.50035	.508	.434	.3395
$\gamma_1 = \gamma_2 = 1.1$	.0585	.136	.508	.519	.434	.345
$\gamma_1 = \gamma_2 = 1.2$	.0528	.121	.512	.530	.435	.350
$\gamma_1 = \gamma_2 = 1.3$	.0476	.108	.517	.540	.436	.352
$\gamma_1 = \gamma_2 = 1.4$	.0429	.096	.521	.550	.436	.354
$\gamma_1 = \gamma_2 = 1.5$	.0387	.085	.525	.559	.436	.356
$\gamma_1 = \gamma_2 = 1.6$	.0348	.0757	.529	.568	.436	.357
$\gamma_1 = \gamma_2 = 1.7$	.0313	.0669	.533	.576	.436	.358
$\gamma_1 = \gamma_2 = 1.8$	.0281	.0592	.537	.581	.435	.360
$\gamma_1 = \gamma_2 = 1.9$	.0252	.0522	.540	.590	.435	.358

Assumptions: $\pi_{00} = \pi_{11} = .75$ $\quad \bar{W} = 1.00 \quad \bar{T} = 2.00$

$\bar{T}_c = 1.00 \quad \bar{T}_\ell = 1.00 \quad R_c^0 = 0 \quad R_c^1 = 1.5$

$R_\ell = 1.0$

(L) - Low current returns to illegal human capital

(H) - High current returns to illegal human capital case individuals devote .15% or 10% of their time depending on whether the current payoff to illegal skills is low or high.

TABLE 2

Fraction of Time Devoted to the Acquisiton of Illegal Skilis

Probability of Current State Continuing	Current State Low Rewards R_c=0	Current State High Rewards R_c=1.5
$\pi\pi_{00} = \pi\pi_{11} = .25$	.085	.036
$\pi\pi_{00} = \pi\pi_{11} = .50$	.065	.065
$\pi\pi_{00} = \pi\pi_{11} = .75$	.036	.085
$\pi\pi_{00} = \pi\pi_{11} = .85$	.025	.009
$\pi\pi_{00} = \pi\pi_{11} = .99$	.0015	.1002

Assumptions: $\bar{W} = 1.00$ $\bar{T} = 2.00$

$\bar{T}_c = 1.00$ $\bar{T}_\ell = 1.00$ $R_c^0 = 0$ $R_c^1 = 1.5$

$R_\ell = 1.00$ $\gamma_1 = \gamma_2 = 1.5$

4. Youth Crime and Adult Crime: A Skills Matching Model.

I. Introduction

In the previous chapters we have developed models that attempt to describe the decision to acquire the skills useful in illegal activities as part of a rational decision process in the face of uncertain payoffs. The results seem broadly consistent with many of the stylized facts about criminal activity and with common sense. One of the more difficult features to explain however is the lifetime profile of individuals' involvement with criminal activity. It is well known and well documented that a very high percentage of youths (male youths in particular) have some experience with crime, while only a small fraction of adults have such an involvement. Similarly, many studies seem to indicate that people are more likely to cheat on their income tax when young than when old. Analyses of the age distribution of people convicted for crimes show a much higher probability that the individual will be under thirty. People seem to outgrow crime except for a small fraction who become career criminals. There are many explanations for this phenomenon from biological to sociological and the true explanation is undoubtedly complex. Our goal here however is to see to what extent this phenomenon could be consistent with a model of rational choice under uncertainty.

The model we examine here is similar in spirit to what are known as matching models in the labor economics literature. These have been developed to explain the phenomenon that there exists a distribution of wage offers in equilibrium and that this can be consistent with optimal job search activity by individuals. Matching models thus introduce heterogeneity into the wage structure without greatly complicating the analysis. Our model views the population as composed of heterogeneous individuals, many of whom may attempt criminal activity in their youth. The true monetary reward from engaging in illegal activity is unknown to individuals when they are young. It is observed with error and the young must make a decision to engage in criminal activity without knowing the true payoff to that activity.

In the second period of life individuals learn about the true payoff to illegal activity for them. The true payoff is a random variable distributed asymmetrically across the population. This implies that there will be only a portion of the population for whom the "match" with illegal activity may be sufficiently lucrative for them to continue when the true value of the match is known. We know that only a small fraction are likely to continue in illegal careers so the distribution of the successful matches reflects the fact that for the majority the opportunities from legal employment will eventually be viewed as dominant.

In the next secton we construct a model of labor force participation that has the features described above. We make a series of assumptions about distributions of the random variables that agents face and then proceed to simulate the model. We find this simple model surprisingly successful at explaining the simple observations it set out to explain. There can be no doubt that the true explanation for lifecycle patterns in criminal activity is more complicated than our simple model reflects but it is interesting that a reasonable model of uncertainty with risk aversion is so successful at reproducing the patterns.

II. The Model.

We assume a population of heterogeneous agents who differ only in their marginal productivity in illegal activities. Individuals differ in their ability to succeed in illegal ventures and the true monetary payoff from an illegal activity is represented by a "match" parameter, θ. For each individual there is a true θ but the match parameters are distributed asymmetrically over the population. We assume the match parameters are distributed lognormally with mean μ and variance σ^2. As in the overlapping generations model of the previous section, it is assumed that agents live for two periods. When they are young the true value of the match parameter (the marginal productivity in illegal activity) is unknown. The agents observe a noisy payoff, $x = \theta \cdot \xi$, where ξ is drawn from a lognormal distribution with mean 0 and unit variance. Based on this noisy payoff the agents have to form the expectation $E(\theta|x)$ of their marginal productivity in illegal endeavors in the future. We assume that the agents form this expectation rationally and know that they will have the true value of their marginal productivity revealed in the next period.

The alternative to illegal activity is to enter the labor market for legal activities. We assume that agents can always find employment in this market at a competitively determined market wage $\bar{W}$. This alternative prevails in the second period of an agent's life as well. Because we have assumed that the marginal productivity in illegal activities becomes known to agents in the second period the implication is that they will then choose employment in the legitimate labor market if $\theta < \bar{W}$ and will continue the illegal activities if $\theta > \bar{W}$.

The individual's decision problem is to maximize expected lifetime utility of consumption:

$$\text{(1)} \qquad \text{Max} \quad E_t \sum_{s=0}^{\infty} \beta^{t+s} \, U(Z_{t+s})$$

subject to the constraints

$$Z_{t+s} = \tilde{W} \text{ for all } s \qquad \text{if agent participates in legal activity}$$

$$Z_{t+s} = \begin{cases} m_1 & s = 0 \\ \theta & \text{for } s > 0 \end{cases} \qquad \text{if agent participates in illegal activity.}$$

Given the assumption about the random variables θ and ξ

$$\log \theta \sim N(\mu, \sigma_\theta^2)$$

$$\log \xi \sim N(0,1)$$

we assume that agents use Bayes' law to calculate the posterior probability distribution of θ given the noise ridden observation $x = \theta \cdot \xi$. From Bayes' rule we know that

(2) $$E(\log \theta | x) = \mu + (\sigma_\theta^2/(1+\sigma_\theta^2)) \cdot \log(x-\mu) = D_1$$

(3) $$\text{var}(\log \theta | x) = \sigma_\theta^2/(1+\sigma_\theta^2) = D_2$$

which implies that

(4) $$m_1 = E(\theta | x) = \exp(D_1 + .5 \cdot D_2)$$

(5) $$\sigma_2^2 = \text{var}(\theta | x) = \exp(2D_1 + 2D_2) - \exp(2D_1 + D_2)$$

These moments provide the information needed for the constraints. The term D_1 is just the conditional expectation of log θ given x and D_2 is the conditional variance of log θ given x.

We can represent a solution to this problem as a dynamic programming problem with recursive structure. Bellman's functional equation for this problem is given by

$$(6) \qquad v(m_1) = \max[U(m_1) + \beta \int^{\infty} J(\theta)\ dF(\theta|x),\ U(\bar{W})/1-\beta]$$

where

F(θ|x) is the condition distribution of θ given x,

$J(\theta) = \max[U(\theta)/1-\beta,\ U(\bar{W})/1-\beta]$

Given the assumptions about θ and ξ equation (6) can be rewritten as

$$(7) \qquad v(m_1) = \max[\{U(m_1) + (\beta/1-\beta) \cdot \int_{\bar{W}}^{\infty} U(\theta)\ dF(\theta|x) + (\beta/1-\beta) \cdot \int_{0}^{\bar{W}} U(\bar{W})dF(\theta|x)\},\ U(\bar{W}/1-\beta)].$$

We proceed by assuming that there exists a reservation return, $\bar{m}_1$, such that an agent will decide to get involved in an illegal venture if his expected payoff m_1 exceeds $\bar{m}_1$, otherwise he participates in the labor market for the riskless wage $\bar{W}$.

Using the value function in (7) this reservation return, $\bar{m}_1$, is given by the root of the following integral equation:

$$(8) \qquad U(m_1) + (\beta/1-\beta) \cdot \int_{\bar{W}}^{\infty} U(\theta)\ dF(\theta|m_1,\sigma_1^2) + (\beta/1-\beta) \cdot U(\bar{W}) \int_{0}^{\bar{W}} dF(\theta|m_1,\sigma_1^2) = U(\bar{W}/1-\beta)].$$

Our interest lies in analyzing the probabilities of the following two events: (i) An agent will commit an offense in his young age (at date t) and quit it at date t+1 forever; (ii) he will commit an offense in his young age and choose not to quit in his old age. These probabilities can be represented by:

$$(9) \qquad \text{Prob}(\theta < \bar{W};\ m_1 > \bar{m}_1) = \int_0^{\bar{W}} \int_{m_1}^{\infty} dF(\theta | m_1, \sigma_1^2)\ dG(m_1)$$

$$(10) \qquad \text{Prob}(\theta > \bar{W};\ m_1 > \bar{m}_1) = \int_{\bar{W}}^{\infty} \int_{m_1}^{\infty} dF(\theta | m_1, \sigma_1^2)\ dG(m_1)$$

where $G(m_1)$ is the unconditional distribution function of m_1.

Note that m_1 itself is also distributed lognormally with mean, μ_2 and variance, σ_2^2 given by:

$$(11) \qquad \mu_2 = \exp(\mu + \tfrac{1}{2}\ (\sigma_0^2/1 + \sigma_0^2) + \tfrac{1}{2} \cdot (\sigma_0^4/1 + \sigma_0^2))$$

$$(12) \qquad \sigma_2^2 = \exp(2\mu + \sigma_0^2/1 + \sigma_0^2) \cdot [\exp(2\sigma_0^4/(1 + \sigma_0^2)) - \exp(\sigma_0^4/(1 + \sigma_0^2))]$$

The details of the derivation are presented in Appendix 1.

In general, these probabilities will depend on the riskless wage, $\bar{W}$, and the preference parameters involving the discount factor, β, and the curvature of the utility function, $U(\cdot)$. The model with its present level of generality is not useful for prediction purposes. In the following section, the model is calibrated for a particular choice of $U(\cdot)$.

III. A Calibration Experiment.

We assume that the utility function has a constant relative risk aversion form $U(Z) = Z^{1-\gamma}/1-\gamma$ where $0 < \gamma < \infty$. The equation (8) then reduces to

$$m_1^{1-\gamma}/1-\gamma + \beta \cdot \int_{\bar{W}}^{\infty} \theta^{1-\gamma}/(1-\beta)(1-\gamma) \cdot dF(\theta|m_1,\sigma_1^2)$$

(13)

$$- \beta\, \bar{W}^{1-\gamma}/(1-\beta)(1-\gamma)\ [1-\int_0 dF(\theta|m_1,\sigma_1^2)] = 0.$$

Equation 13 is a non-linear integral equation in m_1. To get a solution to the model this function is computed for different grids of m_1 to search for its zero value. The details of the numerical methods used in the solution are discussed in Appendix 2.

There are three parameters in this model, namely, β, γ and $\bar{W}$. The value of β is chosen to be 0.96 which conforms with an observed dividend/price ratio of .0496 as obtained by Shiller (1981). Arrow (1971) summarizes a number of studies and concludes that the relative risk aversion parameter, γ is approximately constant and in the neighborhood of 1.00. Friend and Blume (1975) presents evidence based on portfolio holdings of individuals in favor of large value of γ in the neighborhood of 2.00. Kydland & Prescott (1982) found a value between one and two to reproduce the observed relative variabilities of consumption and investment. Tobin and Dolde (1971) studied life cycle savings behavior with borrowing constrains and used a value 1.5 to fit the observed life cycle savings pattern. For practical purposes, we simulated our model for grids of γ values within the range (1,2). The final parameter, $\bar{W}$, is the monetary reward from riskless occupation. The value of $\bar{W}$ is chosen to be 3.5 which conforms with the existing legal minimum wage in the U.S. economy.

Table 1 reports the reservation return $\bar{m}_1$ and the relevant probabilities for different values of γ.

The numbers in Table 1 illustrate the probability that agents will engage in two alternative lifetime employment paths. On one path individuals try the illegal activity when young and then revert to legal endeavors when old. On the other they sample illegal activity when young and stick to it. The probabilities of the first path are illustrated by the third column of Table 1, while the probabilities of the second path are illustrated by the fourth column. The probabilities are arranged according to increasing values of the measure of risk aversion. The absolute value of the reservation wage m_1 are meaningless per se but the behavior with changes in the coefficient of relative risk aversion are significant.

The results in Table 1 reveal the following characteristics of labor supply in this simple economy:

1. The reservation return m_1, the reservation conditional expectation necessary to induce participation in illegal activity increases as the relative risk aversion increases.

2. For reasonable parameter values the probability of engaging in illegal activities when young can be extremely high even while the probability of engaging in such activities when old is extremely low.

3. The probability of engaging in illegal activity when young decreases sharply with increases in relative risk aversion while the probability of continuing in that path changes only slightly.

TABLE 1

Effect of Change in the Relative Risk Aversion

$\bar{W} = 3.5, \quad \beta = 0.96$

γ	$\bar{m}_1$	$Prob(\theta < \bar{W};\ m_1 > \bar{m}_1)$	$Prob(\theta > \bar{W};\ m_1 > \bar{m}_1)$
1.2	0.745	0.7119	0.0492
1.5	1.14	0.5043	0.0443
1.7	1.335	0.4188	0.0411
1.9	1.495	0.3583	0.0383
2.0	1.56	0.3345	0.0371

These results all seem consistent with the observed characteristics we set out to explain. There can be a high propensity to sample from the risky and unknown wage distribution when young, even when the probability of profiting from that choice over a lifetime is very small. Again it is worth emphasizing that these dramatically different probabilities are simply the result of uncertainty and not the result of irrationality or aberrant behavior in any form.

Table 2 presents the results of further simulations that are designed to test the sensitivity of these results to changes in the absolute magnitude of the riskless wage W that is meant to capture the return to legal activity. In these simulations we fixed the relative risk aversion parameter at 1.5 while varying W. Column 3 again contains the probability of engaging in illegal activity only when young while column 4 lists the probabilities of engaging in those activities throughout one's life. Table 2 shows that:

1. The reservation return increases monotonically in W. This implies that the better the financial rewards for legal employment the higher the expected gain from illegal activities would have to be to induce individuals to engage in them.

2. The probability of engaging in illegal activity just once is monotonically decreasing in W, the return to legal activity.

3. The probability of adopting a career of illegal activity is also monotonically decreasing in W.

Riskless Wage	$> \bar{m}_1)$	$Prob(\theta > \bar{W} \quad m_1 > \bar{m}_1)$
0.96		0.0443
		0.0394
		0.0304

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3. As the degree of risk aversion increases, individuals substitute away from the acquisition of illegal human capital towards the acquisition of legal human capital.

4. The existence of uncertainty implies that under reasonable assumptions for economic conditions facing young men, a large fraction of the youth population will "experiment" with criminal activities, but that this behavior will be "outgrown" very quickly. Our simulations, for example, predict that sice the payoffs to criminal activities are unknown, between 30 and 70 percent of the youth population will commit some crimes, but will refrain from criminal activities after this "trial period". In addition, a relatively robust (to alternative parametric assumptions) 3-4 percent of the youth population will find it profitable to commit crimes while young, and remain in a criminal career for the remainder of the working life.

These predictions show that the theoretical approach we present in this paper can substantially increase our understanding of criminal behavior or, more generally, of behavior in markets characterized by a large degree of uncertainty about payoffs to specific activities. The static price-theoretic labor supply models which fill the literature can yield essentially only one theorem: the higher the cost of committing criminal activities, the lower the probability that a person will undertake that type of activity. The asset-pricing (and matching) models developed in this report provide a major expansion of the predictive power and scope of economic theory in the understanding of criminal behavior.

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Appendix 1

Proof of Equations (11) and (12)

Taking log transform then taking expectation operator, we get

$$E(\log m_1) = E(D_1) + (0.5)\ E(D_2)$$
$$= E[\mu + (\sigma_0^2/1 + \sigma_0^2)\ (\log x - \mu) + (0.5) \cdot E(\sigma_0^2/1 + \sigma_0^2)$$
$$= \mu + (0.5) \cdot (\sigma_0^2/1 + \sigma_0^2)$$
$$= J_1 \text{ (say).}$$

because

$$E(\log x) = E(\log \theta) + E(\log \xi) = \mu.$$

Similarly,

$$\text{var}(\log m_1) = \text{var}(D_1 + 0.5D_2) = \text{var}(D_1)$$
$$= \text{var}(\mu + \sigma_0^2/(1 + \sigma_0^2) + (\log x - \mu))$$
$$\sigma_0^4/1 + \sigma_0^2 \cdot = J_2 \text{ (say).}$$

Hence, we have proved that $\log m_1 \sim N(J_1, J_2)$.

Which implies

$$E(m_1) = e^{J_1 + 0.5J_2} = \mu_2 \text{ (say)}$$

and $\text{var}(m_1) = e^{2J_1 + 2J_2} - e^{2J_1 + J_2} = \sigma_2$ (say)

thus proving the result.

Q.E.D.

Appendix 2

In this appendix, the details of the numerical solution method are described.

Step-1: Computation of reservation return, $\bar{m}_1$

We started with a wild guess of the root $\bar{m}_1$ and then examined whether the function in equation (13) behaves monotonically when $\bar{m}_1$ is increased by a very small step size. For our choice of parameter values β, γ, $\bar{W}$, fortunately this function displays monotonicity and hence, it admits of a unique root.

The integral involved in equation (13) is computed by using Simpson's approximation principle which is available as a gauss sub-routine called "Intsimp." Using Chebyshev's rule mean plus three times standard deviation is taken as a rough estimate of infinity.

Step-2: Computation of joint probabilities (9) and (10).

Each of these probabilities involves a double integral. The following approximation method is used. Note that we can rewrite the joint probability in equation (9) as:

$$\text{Prob}(\theta < \bar{W};\ m_1 > \bar{m}_1) = \int_{m_1}^{\infty} \text{Prob}(\log\theta < \log \bar{W} | m_1) \cdot f(m_1)dm_1.$$

where $f(m_1)$ is the density function of m_1.

Using the fact that m_1 is log normal, (9) reduces to:

$\text{Prob}(\theta < \bar{W};\ m_1 > \bar{m}_1) =$

$$\int_{m_1}^{\infty} \text{Prob}(\log \theta < \log \bar{W} | m_1) \cdot \frac{1}{\sqrt{2\pi}\sigma_2 m_1} \cdot e^{-\frac{1}{2\sigma_2^2}(\log m_1 - \mu_2)^2} dm_1$$

where $\mu_2 = E(m_1)$

$\sigma_2^2 = \text{var}(m_1)$.

As a discrete approximation, (9) can be written as:

$$\text{Prob}(\theta < \bar{W};\ m_1 > \bar{m}_1) = \sum^{\infty} \text{Prob}(\log \theta < \log \bar{W} | m_1^i).$$

which can be computed by taking fine grids Δm_1^i. We have chosen Δm_1^i around .001. Taking finer grids than this does not change the result significantly.